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# Generation portfolio optimization under wind Production Tax Credit and Renewable Portfolio Standard

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**Generation portfolio optimization under wind Production Tax Credit and  
Renewable Portfolio Standard**

by

**Chenlu Lou**

A thesis submitted to the graduate faculty  
in partial fulfillment of the requirements for the degree of  
**MASTER OF SCIENCE**

Major: Industrial Engineering

Program of Study Committee:

Jo Min, Major Professor

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Ames, Iowa

2011

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## ABSTRACT

In this thesis we construct and analyze a mean-variance utility maximization model for a risk-averse electric power generation company who wishes to determine the optimal levels of capacity and production from a single conventional fuel source and wind energy subject to the state Renewable Portfolio Standard (RPS). We assume the conventional fuel price and the federal wind power Production Tax Credit (PTC) level are random variables. This study is motivated by the highly stochastic nature of the PTC level and the existing competing claims for the impact of the RPS on the renewable energy development. Throughout our model we show how vastly different arguments and claims for the PTC and RPS policy can be accommodated within a single framework. We also analytically and numerically show how the RPS level, standard deviations of the fuel price and PTC level and their correlation coefficient would affect the generation company's decisions. Interesting and relevant managerial insights and economic implications are presented, as well as policy guidelines and recommendations for the PTC and RPS.



## CHAPTER 1. INTRODUCTION

### 1.1 Background

Due to the increasing public concerns over climate change and air pollution, renewable technologies of electricity generation have been promoted all over the world in recent years. Sources of renewable electricity generation includes wind, solar, geothermal energy *etc.*, among which wind plays a most significant role. By the end of 2009, the worldwide wind capacity has reached 159,213 MW, out of which 38,312 MW were added in 2009 (WWEA, 2010).

According to EIA's report of Electric Power Industry 2009 (2010), in the U.S, about 14,650 MW of wind power capacity has been installed in 2009, which represents 63.3% of all the new installed U.S. capacity for that year. The booming development of wind generation capacity in U.S. have been accelerated driven by governmental policies and subsidies that encourage generation from renewable energy sources. Among the most significant incentives of this growth are the federal Production Tax Credit (PTC) and a series of state Renewable Portfolio Standards (RPS).

#### 1.1.1 Overview of the Federal PTC

Established by the Energy Policy Act of 1992, the PTC is designed to stimulate the development of renewable energy by providing a production-based credit for the first 10 years of project options (Wiser *et al.*, 2007). The initial value of the PTC was 1.5 cent per kilowatt-hour, which is designed to be adjusted upwards in future years for inflation. Since first enacted, the PTC has been renewed or extended on several occasions. Currently the

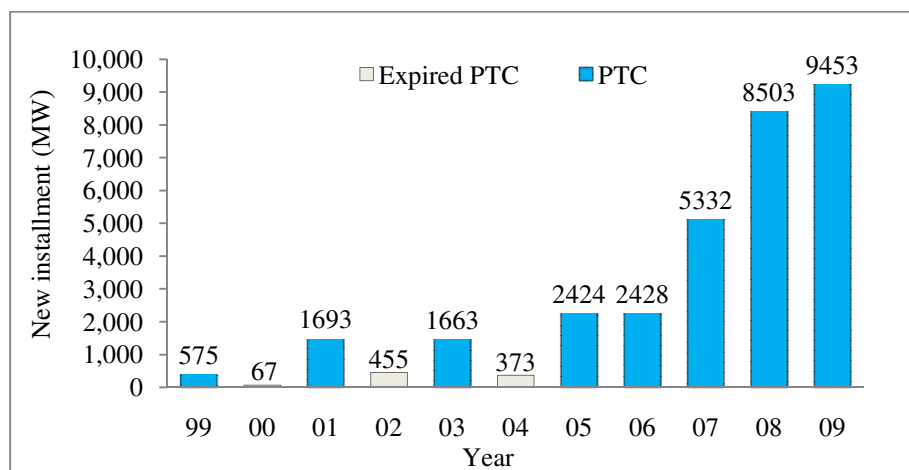
value of PTC for wind is 2.2 cent per kWh, which was extended to the end of 2012. The following table summarizes the legislative history of the PTC.

**Table 1. Legislative history of the PTC**

|   | Legislation   | Date enacted | PTC eligibility window | PTC lapse duration | Effective duration                     |
|---|---|--------------|------------------------|--------------------|--|
| 1 | Section 1914, Energy policy Act of 1992                                 | 10/24/92     | 11/01/92-06/30/99      | N/A                | 11/01/92-06/30/99                      |
| 2 | Section 507, Ticket to Work and Work Incentives Improvement Act of 1999 | 12/19/99     | 07/01/99-12/21/01      | 01/01/00-06/30/00  | 07/01/99-12/31/99<br>07/01/00-12/31/01 |
| 3 | Section 603, Job Creation and Worker Assistance Act                     | 03/09/02     | 01/01/02-12/31/03      | 01/01/02-02/28/02  | 03/01/02-12/31/03                      |
| 4 | Section 313, The working Families Tax Relief Act                        | 10/04/04     | 01/01/04-12/31/05      | 01/01/04-09/30/04  | 10/01/04-12/31/05                      |
| 5 | Section 1301, Energy policy Act of 2005                                 | 08/08/05     | 01/01/06-12/31/07      | N/A                | 01/01/06-12/31/07                      |
| 6 | Section 201, Tax relief and Health Care Act of 2006                     | 12/20/06     | 01/01/08-12/31/08      | N/A                | 01/01/08-12/31/08                      |
| 7 | Section 101&102, The energy Improvement and Extension Act of 2008       | 10/03/08     | 01/01/09-12/31/09      | N/A                | 01/01/09-12/31/09                      |
| 8 | Section 1101&1102, The American Recovery and Reinvestment Act of 2009   | 02/17/09     | 01/01/10-12/31/12      | N/A                | 01/01/10-12/31/12                      |

As one of the biggest drivers of the growth of wind capacity in U.S, the PTC helps to reduce the price of wind-generated electricity, which even makes wind power now economically attractive in some regions. Its impact on wind generation expansion can also be partially observed by the fluctuation of annual installed capacity of wind since the PTC was

first enacted in 1992. The annual new installed capacity of wind turbines is showed in the following Figure 1.



**Figure 1. Annual new installed capacity of wind turbines in U.S. (1999-2009)**

Specifically, the PTC expired in 2000, 2002 and 2004, each time resulting in decreases of at least 50% in new installed capacity (DOE, 2006). This historical experience indicates that the frequent expiration/extension cycle of PTC might have negative consequences for the growth of wind power. This boom-and-bust cycle might have made the PTC less effective in stimulating low-cost wind development for the reason that potential investors self-select to avoid investment in wind power due to the uncertainty in PTC.

Another example of the negative effects of tax incentive expiration can be observed in the biodiesel industry in the U.S. The biodiesel production has dropped to almost zero since the biodiesel tax credit expired in 2009 (Gelsi, 2010).

Since the current PTC will be expired by the end of 2012, no one knows what the level of it would be for 2013 addition. According to AWEA's Third Quarter 2010 Market Report (2010), U.S. added 395 megawatts (MW) of wind power in the third quarter of 2010,

72 percent below this time last year. A higher level PTC with less uncertainty might have mitigated this drop.

### **1.1.2 Overview of the State RPSs**

In combination with the PTC, state RPS requirements have also emerged as one of the most important drivers of renewable energy capacity additions (Wiser and Barbose, 2008). It requires electric utilities and other retail electric providers to supply a specified minimum amount of customer load with electricity (typically as a percentage of total supply) from eligible renewable energy sources. The goal of an RPS is to stimulate market and technology development so that, ultimately, renewable energy will be economically competitive with conventional forms of electric power (EPA, 2009).

Mandatory RPS policies are backed by various types of compliance enforcement mechanisms, and many such policies include the trading of renewable energy certificates (RECs) (Wiser and Barbose, 2008). The use of RECs increases compliance flexibility and may therefore reduce overall compliance costs.

Currently 29 states and DC as well as Puerto Rico have adopted the RPS, which is illustrated by the following map in Figure 2 (DSIRE, 2010). RPS policies have also been developed in several other countries, and have been considered (but not adopted) by the U.S. Congress (Chen *et al.*, 2007).

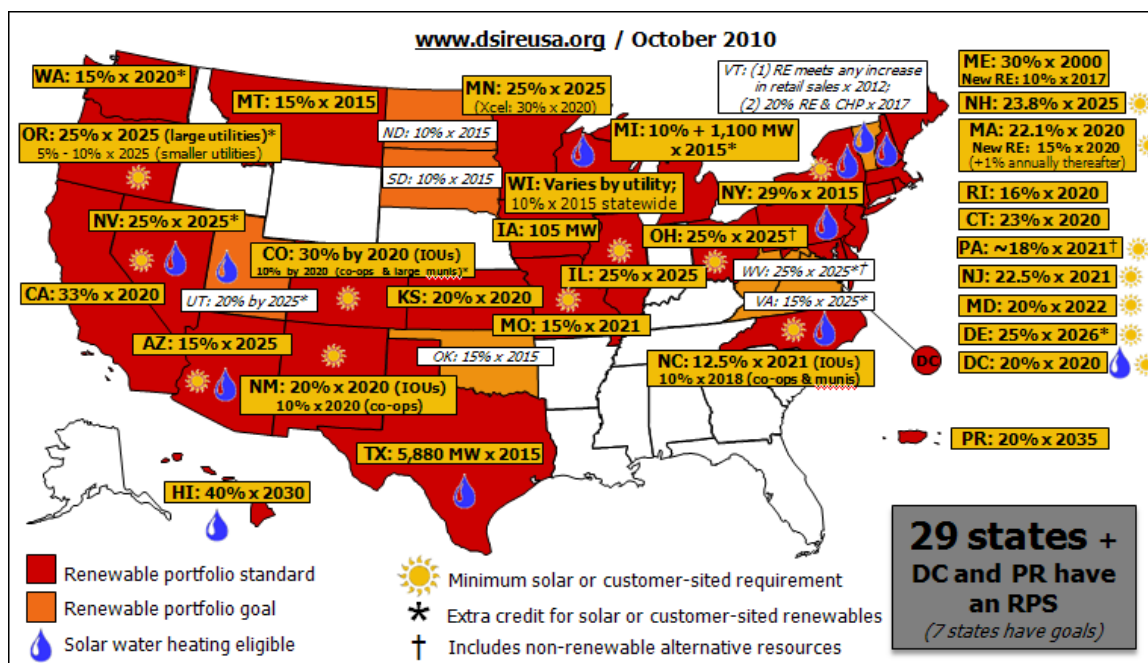


Figure 2. Map of current RPS adoptions in U.S. (2010)

Typically, each region adopts an RPS for various objectives, which include ensuring an adequate supply of electric power when conventional imported fuel prices are high. This mechanism is often viewed as a cost-effective, market-based approach that is administratively efficient. According to a technical report from the Berkeley Lab (Chen *et al.*, 2007), the long-term rate impacts of state RPS policies are projected to be relatively modest in most cases: for most of the studies in their sample the predicted rate increases are no greater than 1%. However, some other groups claim that the RPS would increase near term electricity costs and might lead to little or no change (Ashton, 2008).

As mentioned before, U.S. Congress is considering legislation of establishing a national RPS, which raised much debate about its potential merits and hazards (Joshua, 2008). For example, some researchers warn that a national RPS is not practical and might lead to blackouts (see Apt *et al.*, 2008).

At this point of time, however, it is absolutely unclear if these contradictory statements are due to the fundamental differences in their perspectives which manifest in vastly different sets of initial assumptions, or in the policy management and operations within a single framework of a perspective under the same set of assumptions.

## 1.2 Objective

The objective of this study is to construct an analytical model that concurrently address PTC and RPS policy issues within a single framework and accommodate vastly different arguments and claims about these issues. Based on the analyses, we will also provide managerial insights and economic implications, as well as policy guidelines and recommendations that are quite relevant to practitioners and academics. We hope this study could help analytically comprehend the roles and examine the effectiveness of the PTC and RPS on the generation expansion planning and renewable energy. Here the policy effectiveness is defined as a performance assessment, i.e., it judges how the actual effect of the policy measure up to its objective (UNEP, 2009).

We formulate a mathematical model for a risk-averse electric power generation company that wishes to determine the optimal levels of capacity and production from a conventional power source (e.g., natural gas and coal) as well as wind energy under the RPS regulation when the conventional fuel price and wind power PTC are random variables.

For this study, we utilize mean-variance (MV) utility function to account for the generation company's risk-averse nature. First raised in 1952, the mean-variance portfolio theory (MVP) has been developed and widely used in portfolio optimization (Markowitz, 1952). It optimizes the portfolio's utility by defining the trade-off between the expected

profit (or return) and variance of the profit (or return) (Liu and Wu, 2006). Parametric analysis is performed on each of the critical factors, i.e., the RPS minimum requirement, the standard deviations of the conventional fuel price and wind power PTC as well as their correlation coefficient.

### 1.3 Thesis Organization

The rest of the thesis is organized as following: In Chapter 2 we present a comprehensive review of the previous literatures on power generation portfolio optimization as well as relative renewable energy policy research. In Chapter 3 we mathematically formulate the utility maximization problem for the generation company by employing the MVP theory collaborated with the PTC and RPS. Optimality conditions will also be derived. In Chapter 4 we conduct parametric analyses on each critical parameter to show how they affect the generation company's decisions. Chapter 5 provides a numerical example to illustrate our insights obtained from the previous chapters. Relevant policy implications are discussed and summarized in Chapter 6. In Chapter 7 the preliminary investigation given a negative correlation coefficient between the conventional fuel price and wind PTC is presented. Conclusion and future work will be provided in Chapter 8.

## CHAPTER 2. LITERATURE REVIEW

As for the generation expansion planning (GEP), traditional planning has been studied for many years. Kagianas *et al.* (2004) summarizes the traditional modeling techniques for GEP under monopoly. Mathematically, to solve a GEP optimization problem is equivalent to find a set of optimal decision vectors that minimizes an objective function under multiple constraints. Among all the methods that have been used to solving this problem computationally, dynamic programming (DP) is one of the most significant and widely used algorithms. Other used methods include Bender's decomposition algorithm, linear approximation and Genetic Algorithm, *etc.*

In particular, Booth (1972) formulates the long-term generation expansion problem by combining a method of production costing based on probabilistic simulation methods with an advanced DP problem formulation. The formulated problem can be solved by an "Open Loop Feedback" approach which is developed by P. H. Henault. Considering all the system states shown to be feasible, this paper also effectively reduces the dimensionality of the problem, which allows the rapid calculation of the optimal generation expansion plan.

On the other hand, due to the transformation from monopoly to competition in the electric power market, game theory has also been adopted to formulate GEP problem. Chuang *et al.* (2001) utilizes the Cournot theory of oligopoly to model GEP in a competitive electric power industry. In this model, each firm independently decides on its capacity reserve participation level, and the generating units are tradable through bidding. Throughout numerical experiments it points that the industry expansion and system reliability under Cournot competition are greater than those under centralized expansion planning.



In recent years, there have been more efforts to exploit relatively sophisticated economic models under uncertainty for such planning. As we mentioned in Chapter 1, the economic balance between the mean and variance has also been extensively studied in the electric power management and operations literature. Wang and Min (2008) formulates an electric power portfolio model which aims to maximize the profit while minimize the financial risk of the portfolio. It is able to quantify profits and financial risks in a single framework which takes account for the risks of forced outages as well as price differentials between day-ahead and real-time markets.

Due to the booming development of renewable energy all over the world, GEP including the renewable energy sources has also received much research attention in the recent years. Uncertainties in the fuel and electricity price, renewable energy reliability and governmental policies have been studied under MV framework in some literature.

Roques *et al.* (2008) adopts a MVP model to identify optimal base load generation portfolios for large generators in a liberalized electric power market. It uses Monte Carlo simulation to generate a series of results of gas, coal, and nuclear plant investment returns as inputs. The results turn to show that a high degree of correlation between gas and electricity prices reduces gas plant risks and make portfolios dominated by gas plant more attractive.

Doherty *et al.* (2006) formulates a cost minimization model which combines with wind generation characteristics and generation adequacy. By computationally solving the problem and obtaining least-cost generation portfolio results for different wind penetrations, they analyze the effect of increasing wind capacity and the role of wind generation in least-generation portfolio. MVP is also adopted to analyze the fuel-related electricity cost volatility. Their results show that for a large range of scenarios, wind generation played a

significant role in desirable generation portfolios that are diversified to reduce exposure to fuel price risk.

Awerbuch (2006) is another work utilizing MVP to illustrate how mixed electricity generation can benefit from additional shares of wind, geothermal and other renewables. They first compare the “least cost” and portfolio-based approaches in generation planning and then describe essential portfolio-theory ideas which is used to analyze the effect of fixed-cost technologies (such as renewables) on the generating mix and its implication for energy security. Three portfolio case studies for the EU, U.S. and Mexico are presented to illustrate their methodology. In their analysis, they assume the costs of wind, solar and other capital-intensive renewable are relatively fixed over time and the risk of a generation portfolio comes from the fuel and other generating costs. Specifically for the U.S. case, diversity analysis is also performed due to the drawback on applying historic cost variance and covariance data to the unpredictable future. They finally show that the typical gas-coal generation portfolio offers little diversification and provides little insulation from the systematic risk of coal and gas price movements.

Liu and Wu (2006) proposes a sequential approach to optimize electric energy allocation for a generation company based on the MVP. They define a risk-penalty factor as the term which identifies the company’s degree of risk aversion. This factor could be used in the utility function of the tradeoff between the expected profit and risk of the company which is called as the “profit approach” instead of the common “return approach”. Based on this, the paper develops an analytical and quantitative approach to determine the optimal allocation between spot and contract market, with consideration of fuel price, electricity price as well as congestion charge. The simulation results shows that a higher correlation between

fuel spot price and electricity spot price leads to more electricity traded in the market, which is obviously consistent with intuition and thus helpful to such a company to make trading decisions. Some other relative study includes Liu and Wu (2007).

Now we proceed to review the literature on the policy issues of electric power generation industry. As the most significant drivers of the renewable energy in U.S., PTC and RPS have also been studied in some literature. One of the initial discussions about the effectiveness of PTC mechanism is Kahn (1996), which claims that the impact of PTC will be minimal because it inadvertently raises financing costs. It focuses on project financing as the financing method for developing wind turbines and investigates the capital structure and debt based on it. It also suggests the potential benefits of transforming the wind subsidy from a tax basis to a cash basis.

Barradale (2008) points out that the volatility of wind power investment associated with the PTC is unrelated to the underlying economics of wind. Instead it is due to the dynamics of power purchase agreement (PPA) negotiations which is designated to deal with PTC renewal uncertainty. It also suggests and compares some alternative incentives for encouraging wind industry, such as depreciation rules, pricing or tariff mechanisms and RPSs *etc.*

Comparing to the PTC, RPS seems to be more debatable for its rationale. However, few studies have been conducted to evaluate its effectiveness and efficiency. Berry and Jaccard (2001) discusses the major reasons for the growing popularity of the RPS and raise key considerations in designing a RPS, which includes target selection, eligible resources, applicability, flexibility mechanisms and administrative responsibilities. It also presents a relatively comprehensive summary of the worldwide experience of RPS.

Espey (2001) is another paper focusing on the possible impacts of the RPS on different market participants in a deregulated market. Based on the theoretical designing considerations and practical experiences, it discusses the implication of a RPS on generators, utilities, consumers, environmental groups as well as states.

Palmer and Burtraw (2005) develops a model to compare policies which aim to encourage the use of renewable energy. This model, which can predict future generation from different sources under different policy options, finds that the RPS raises electricity prices, lowers total generation, reduces gas-fired generation, and lowers carbon emissions. It also claims that the size of all these effects are grows with the stringency of the RPS. One of their case studies also shows that a RPS target between 15% and 20% appears to be proper for the year of 2020. In addition, the RPS appears to be more cost-effective at increasing generation from renewable sources than a renewable energy production credit (REPC).

So far there is few study analytically comprehend the roles of the PTC and RPS in the producer's generation expansion planning concurrently in a single framework. This is also the motivation of my research. Through this thesis, the effect of RPS as well as the volatility of PTC renewal on a risk-averse electric power generation company will be examined in an analytical way by adopting a simplified MV model. Corresponding guidelines and recommendations for policy designing will also be obtained based on the analytical results.

## CHAPTER 3. MODEL FORMULATION

In this section, we first formulate the MV utility maximization model for the risk-averse generation company who wishes to determine the optimal capacity levels from the wind energy and a single conventional fuel source subject to the RPS constraint where the fuel price and the wind power PTC level are random variables. Next, we derive the corresponding solutions that will be utilized in the subsequent parametric analyses. In the rest of this thesis we refer to “the risk-averse generation company” as the producer.

### 3.1 Fundamental Model Assumptions

In order for us to focus on the impacts of the RPS when the PTC level is random on the producer’s decisions for the capacity levels, we make the following fundamental assumptions on the model environment. The purpose of these assumptions is to simplify the producer’s decision environment so as to concentrate on the most relevant factors that will enable us to construct a tractable and sensible model and its solutions, which will be shown to lead to a multiple number of critical and intellectually stimulating policy implications. In this way, we also prevent numerous possible secondary factors from rendering the problem-solving process to be intractable or from unnecessarily obfuscating the critical policy implications.

(1) The model framework is static and the generation planning is on an annual basis (\$/year), which has often been used in the generation planning literature (see Kagiannas *et al.*, 2004). Furthermore, except for the aforementioned random variables, all parameters are deterministic and, if applicable, linearly proportional. Specifically, the fuel cost function for

generation from the conventional source is a linear function of generation quantity, and the heat rate is a constant. Also, the annualized capital costs (% for a year) for the conventional fuel source power plant and the wind farm are deterministic and static (Chuang, *et al.*, 2001). These sorts of static, deterministic, and linearly proportional (if applicable) assumptions have been widely used in the generation planning literature (see Doherty *et al.*, 2006).

(2) The capacity factors, which are the ratio of the actual delivered power to the theoretical maximum power (Boccard, 2008), as well as the annual total number of hours of generation are fixed for the conventional fuel source power plant as well as the wind farm. This implies that the determination of the optimal generation quantity is equivalent to the determination of the capacity level. Hence, all implications on one can directly lead to the other, and vice versa.

(3) For the producer's decision making process, the per unit electric power selling price (\$/MWh) will be treated as a static and deterministic parameter for the generation planning purposes. With this assumption, we imply that this price is not under the direct control of the producer, and the producer can treat it as a given quantity. This assumption is applicable to a multiple number of scenarios. For example, there often exist opportunities for long term power purchase agreements (PPA) (Ferrey, 2004), and the producer wishes to deliberate the capacity levels considering a PPA at a particular price level. Or, for an economic feasibility study of a generation portfolio, the producer aims to explore baseline capacity levels after supposing a given price level for decision support purposes.

(4) The means, standard deviations, and the correlation coefficient of the conventional fuel price and wind power PTC level are known to the producer before he/she makes the decisions on the capacity levels (but there is no requirement for him/her to know

the distributions). Also, the correlation coefficient is positive. This is based on our observations as follows. Quantitatively, an annual increase in the PTC level is built in the federal PTC legislation according to a cost index (e.g., the inflation rate) (Wiser *et al.*, 2007). Qualitatively, a higher conventional fuel price tends to encourage policy makers to take more favorable legislative actions on the promotion of the renewable energy (Behrens and Glover, 2008).

### 3.2 Notations

Here we give the notations we use in the following of thesis.

$NE_p$  Net earnings after tax of the generation portfolio including the PTC benefit (\$/year)

$NE_p^0$  Net earnings before tax of the generation portfolio (\$/year)

$NE_c^0$  Net earnings before tax from the conventional source (\$/year)

$NE_w^0$  Net earnings before tax from the wind energy (\$/year)

$R$  Risk aversion factor ( $R > 0$ )

$TPTC$  Benefit from the PTC (\$/year)

$x_c$  Generation quantity from the conventional source (MWh/year)

$x_w$  Generation quantity level from the wind energy (MWh/year)

$p$  Electricity selling price (\$/MWh)

$f$  Conventional fuel price with mean  $F$  and standard deviation  $\sigma_f$  (\$/MBtu)

$s$  PTC level with mean  $S$  and standard deviation  $\sigma_s$  (\$/MWh)

$\rho_{FS}$  Correlation coefficient between  $f$  and  $s$

$\delta$  Minimum RPS requirement of the renewable energy over the total energy (%)

|        |  |
|--------|--|
| $h$    | Heat rate of the conventional power plant (Mbtu/MWh)                     |
| $m_c$  | Maintenance cost of the conventional power generation (\$/MWh)           |
| $m_w$  | Maintenance cost of the wind power generation (\$/MWh)                   |
| $d_c$  | Annualized capital cost of the conventional power plant (%)              |
| $d_w$  | Annualized capital cost of the wind farm (%)                             |
| $c_c$  | Cost of installing unit capacity of the conventional power plant (\$/MW) |
| $c_w$  | Cost of installing unit capacity of the wind farm (\$/MW)                |
| $k_c$  | Capacity factor of the conventional power generation (%)                 |
| $k_w$  | Capacity factor of the wind power generation (%)                         |
| $K_c$  | Installed capacity of conventional power generation (MW)                 |
| $K_w$  | Installed capacity of wind power generation (MW)                         |
| $\tau$ | Corporate income tax rate (%)  |
| $t$    | Annual total number of generation hours (hour/year)                      |

### 3.3 Technical Assumptions

Before we present the complete derivation of the model and its solutions, three technical assumptions are also presented which allow us to focus on the most interesting and relevant cases in practice without considering cases that are theoretically pathological and irrelevant in practice.

Assumption 1: At the optimality, the generation quantities  $x_c$  and  $x_w$ , and the corresponding utility level are all positive.



Assumption 2: The net earnings from the conventional source are positive and sufficiently large, which guarantees that the producer does have a sufficient level of tax liability to take full advantage of the PTC in the ranges of random variables being considered.

Assumption 3: The expected value of the sum of the net earnings before tax from the wind energy and the benefit from the PTC are positive while the net earnings before tax from the wind energy alone can be negative. This is consistent with a critical intent of the PTC. Namely, at least on average, one should not expect to lose money even after the benefit from the PTC.

Under these simplifying but critical assumptions, the MV utility maximization model will be presented in detail in the next subsection 3.4.

### 3.4 Utility Maximization Formulation

Specifically the general frame of our MV utility function is represented as the difference between the expected net earnings of the generation portfolio and the variance of the net earnings multiplied by a risk aversion factor as following

$$\max_{x_c, x_w} E(NE_p) - \frac{1}{2} R \text{var}(NE_p) \quad (3.1)$$

In (3.1),  $NE_p$  designates the net earnings of the generation portfolio consisting of the conventional power plant and wind farm including the benefit from the PTC.  $E(NE_p)$  and  $\text{var}(NE_p)$  are the expected value and variance of  $NE_p$  respectively, and  $R$  is the risk aversion factor which indicates the producer's degree of the risk aversion ( $R > 0$ ), i.e., a higher level of  $R$  indicates a higher level of risk aversion (see Liu and Wu, 2006).

Meanwhile,  $x_c$  and  $x_w$  are the decision variables indicating the optimal generation quantities as we notated before.

In order to derive the expression of  $NE_p$ , we first obtain the mathematical expression of  $NE_p^0$  which consists of two parts:  $NE_c^0$  and  $NE_w^0$ . Hence, the portfolio's net earnings before tax are given by

$$NE_p^0 = NE_c^0 + NE_w^0 \quad (3.2)$$

Let us now proceed to elaborate on each term of (3.2). The revenue from the conventional generation is given by  $p \cdot x_c$  while the fuel and maintenance costs are given by  $f \cdot h \cdot x_c$  and  $m_c \cdot x_c$  respectively. The capacity cost of the conventional power plant is expressed as  $d_c \cdot c_c \cdot K_c$ . Hence  $NE_c^0$  can be formulated as

$$NE_c^0 = (p - fh - m_c)x_c - d_c c_c K_c \quad (3.3)$$

Similarly,  $NE_w^0$  can be attained as

$$NE_w^0 = (p - m_w)x_w - d_w c_w K_w \quad (3.4)$$

Therefore  $NE_p$  can be expressed as

$$NE_p = (1 - \tau)NE_p^0 + TPTC \quad (3.5)$$

The first term of (3.5) can be easily obtained from (3.3) and (3.4), and the second term of  $TPTC$ , which denotes the total benefit from the PTC, is given by

$$TPTC = sx_w \quad (3.6)$$

We also recall that based on Fundamental Model Assumption 2, the installed capacity levels of the conventional power plant and wind farm can be directly expressed in terms of  $x_c$  and  $x_w$  by utilizing capacity factors  $k_c, k_w$  as well as  $t$ . That is

$$K_c = \frac{x_c}{k_c \cdot t} \quad (3.7)$$

$$K_w = \frac{x_w}{k_w \cdot t} \quad (3.8)$$

Utilizing (3.2) through (3.8) we obtain the expression of the net earnings after tax of the generation portfolio consisting of the conventional power plant and wind farm including the benefit of PTC as

$$NE_p = (1-\tau) \left[ (p - fh - m_c)x_c - d_c c_c \frac{x_c}{k_c t} \right] + (1-\tau) \left[ (p - m_w)x_w - d_w c_w \frac{x_w}{k_w t} \right] + sx_w \quad (3.9)$$

The coefficients associated with the decision variables  $x_c$  and  $x_w$  are simplified as

follows: Let  $g = (1-\tau)(p - fh - m_c - \frac{d_c c_c}{k_c t})$ , the net earnings after tax per MWh from the

conventional source. Similarly, let  $w = (1-\tau)(p - m_w - \frac{d_w c_w}{k_w t})$ . We note that  $w$  represents the

modified net earnings per MWh from the wind energy. If it is positive, i.e.,

$p - m_w - \frac{d_w c_w}{k_w t} > 0$ , then  $w$  equals to the net earnings after tax per MWh from the wind

energy. Therefore, it can be arithmetically verified that  $NE_p = gx_c + (w+s)x_w$ , and the

expected value of  $NE_p$  is now given by

$$E(NE_p) = Gx_c + (W + S)x_w \quad (3.10)$$

where  $G = E(g)$ ,  $W = E(w) = w$  as there is no random variable here, and  $S = E(s)$ . It can be easily seen that  $W + S > 0$  based on Technical Assumption 3.

The corresponding variance term is given by

$$\text{var}(NE_p) = (1-\tau)^2 h^2 \sigma_F^2 x_c^2 - 2(1-\tau)h\sigma_F\sigma_S\rho_{FS}x_c x_w + \sigma_S^2 x_w^2 \quad (3.11)$$

or equivalently

$$\text{var}(NE_p) = \alpha x_c^2 + 2\beta x_c x_w + \gamma x_w^2 \quad (3.12)$$

where  $\alpha = (1-\tau)^2 h^2 \sigma_F^2$ ,  $\beta = -(1-\tau)h\sigma_F\sigma_S\rho_{FS}$  and  $\gamma = \sigma_S^2$ .

We note that in (3.12), the first term  $\alpha x_c^2$  represents the variance of the net earnings after tax from the conventional source, the third term  $\gamma x_w^2$  represents the variance of the benefit from the PTC, and the second term  $2\beta x_c x_w$  is the covariance between the net earnings after tax from the conventional source and the benefit of the PTC.

With both the expected value term and the variance term elaborated, the corresponding MV utility function is now given by

$$U = Gx_c + (W + S)x_w - \frac{1}{2}R(\alpha x_c^2 + 2\beta x_c x_w + \gamma x_w^2) \quad (3.13)$$

With the objective function formulated, we proceed to mathematically characterize the RPS constraint. Let  $\delta$  denote the RPS minimum fraction of the generation quantity from the wind energy over the total generation quantity. Then the RPS constraint is

$$\frac{x_w}{x_c + x_w} \geq \delta \quad (3.14)$$

where  $0 < \delta < 1$  (i.e., the pathological cases of 0 and 1 are excluded).

Under the assumption and derivations presented thus far, a mathematically equivalent optimization problem for the risk-averse producer is obtained as follows:

$$\text{P1: } \min_{x_c, x_w} -Gx_c - (W + S)x_w + \frac{1}{2}R(\alpha x_c^2 + 2\beta x_c x_w + \gamma x_w^2)$$

s.t.

$$\delta x_c - (1 - \delta)x_w \leq 0$$

In the next section we will solve P1 analytically and derive the conditions under which the RPS binding /non-binding solutions will be optimal.

### 3.5 Analytical Solutions

We note that the optimization problem given by P1 is a quadratic programming problem with a linear constraint. As the first step to derive the optimal solutions, the corresponding first order necessary conditions (FONC's) can be attained as follows:

First, the Lagrangian function is

$$L = -G \cdot x_c - (W + S)x_w + \frac{1}{2}R(\alpha x_c^2 + 2\beta x_c x_w + \gamma x_w^2) + \lambda[\delta x_c - (1 - \delta)x_w] \quad (3.15)$$

where  $\lambda$  is the associated multiplier of the RPS constraint.

The corresponding FONC's corresponding to (3.15) are

$$\frac{\partial L}{\partial x_c} = -G + R\alpha x_c + R\beta x_w + \delta\lambda = 0 \quad (3.16)$$

$$\frac{\partial L}{\partial x_w} = -(W + S) + R\beta x_c + R\gamma x_w - (1 - \delta)\lambda = 0 \quad (3.17)$$

$$\delta x_c - (1 - \delta)x_w \leq 0 \quad (3.18)$$

$$\lambda \geq 0 \quad (3.19)$$

$$\lambda[(-1+\delta)x_w + \delta x_c] = 0 \quad (3.20)$$

Equation (3.16) and (3.17) characterized a stationary point where  $L$ 's slope is zero, (3.18) and (3.19) define the primal and dual feasibility conditions respectively, and (3.20) represents the complementary slackness condition.

For the second order sufficient conditions (SOSC's), the Hessian matrix of the Lagrangian function is given by  $\begin{bmatrix} \alpha & \beta \\ \beta & \gamma \end{bmatrix}$ , where  $\alpha$ ,  $\beta$  and  $\gamma$  are defined as before and used in (3.12). Hence, the SOSC's are  $\alpha > 0$  and  $\alpha\gamma - \beta^2 > 0$ .

When  $0 < \rho_{FS} < 1$ ,  $\alpha = (1-\tau)^2 h^2 \sigma_F^2 > 0$ , and  $\alpha\gamma - \beta^2 = (1-\tau)^2 h^2 \sigma_F^2 \sigma_S^2 (1-\rho_{FS}^2) > 0$ , thus the SOSC's are met for all feasible values of  $(x_c, x_w)$  under our assumptions.

We first consider the case where (3.18) is binding, i.e.,  $\delta x_c - (1-\delta)x_w = 0$ . The corresponding optimal solution and the associated multiplier  $\lambda^B$  are attained by

$$x_c^B = \frac{(1-\delta)^2 G + \delta(1-\delta)(W+S)}{R[(1-\delta)^2 \alpha + 2\delta(1-\delta)\beta + \delta^2 \gamma]} \quad (3.21)$$

$$x_w^B = \frac{\delta(1-\delta)G + \delta^2(W+S)}{R[(1-\delta)^2 \alpha + 2\delta(1-\delta)\beta + \delta^2 \gamma]} \quad (3.22)$$

$$\lambda^B = \frac{G - R\alpha x_c^B - R\beta x_w^B}{\delta} \quad (3.23)$$

From the non-negativity requirement on  $\lambda$  in (3.19), after substituting  $x_c^B$  and  $x_w^B$  of (3.21) and (3.22) into (3.23), we obtain

$$\delta[G\gamma - (W+S)\beta] - (1-\delta)[(W+S)\alpha - G\beta] \geq 0 \quad (3.24)$$

It can be proved that  $x_c^B$  and  $x_w^B$  are both positive given  $G > 0$  and  $W + S > 0$  as stated in Technical Assumptions 2 and 3.

On the other hand, if (3.18) is non-binding (i.e.,  $\delta x_c - (1 - \delta)x_w < 0$ ) at the optimal solution, the corresponding value of  $\lambda^{NB}$  is zero, and the corresponding optimal  $x_c$  and  $x_w$  are:

$$x_c^{NB} = \frac{G\gamma - (W + S)\beta}{R(\alpha\gamma - \beta^2)} \quad (3.25)$$

$$x_w^{NB} = \frac{(W + S)\alpha - G\beta}{R(\alpha\gamma - \beta^2)} \quad (3.26)$$

Hence, we note that the condition under which  $(x_c^{NB}, x_w^{NB})$  is optimal is from the RPS constraint

$$\delta[G\gamma - (W + S)\beta] - (1 - \delta)[(W + S)\alpha - G\beta] < 0 \quad (3.27)$$

We note that (3.24) and (3.27) are both conditions about the critical term  $\delta[G\gamma - (W + S)\beta] - (1 - \delta)[(W + S)\alpha - G\beta]$ . From them we have the following two properties:

Property 1: When  $x_c^B$  and  $x_w^B$  are given by (3.21) and (3.22),  $(x_c^B, x_w^B)$  is optimal if and only if (3.24) holds.

Property 2: When  $x_c^{NB}$  and  $x_w^{NB}$  are given by (3.25) and (3.26),  $(x_c^{NB}, x_w^{NB})$  is optimal if and only if (3.27) holds.

Based on the two properties, we summarize the optimal solutions and corresponding conditions in the following table.

**Table 2. Optimal solutions and corresponding conditions for P1**

|                     | Optimal solution   | Condition  |
|---------------------|--|--|
| Binding optimal     | $x_c^B = \frac{(1-\delta)^2 G + \delta(1-\delta)(W+S)}{R[(1-\delta)^2 \alpha + 2\delta(1-\delta)\beta + \delta^2 \gamma]}$ $x_w^B = \frac{\delta(1-\delta)G + \delta^2(W+S)}{R[(1-\delta)^2 \alpha + 2\delta(1-\delta)\beta + \delta^2 \gamma]}$ | $\delta[G\gamma - (W+S)\beta] - (1-\delta)[(W+S)\alpha - G\beta] \geq 0$ |
| Non-binding optimal | $x_c^{NB} = \frac{G\gamma - (W+S)\beta}{R(\alpha\gamma - \beta^2)}$ $x_w^{NB} = \frac{(W+S)\alpha - G\beta}{R(\alpha\gamma - \beta^2)}$  | $\delta[G\gamma - (W+S)\beta] - (1-\delta)[(W+S)\alpha - G\beta] < 0$    |

So far we have characterized the optimal solutions and the corresponding conditions on the parameters. We now proceed to derive expressions of total generation quantities, expected values and variances of  $NE_p$ , and the corresponding portfolio's utility levels.

First we denote the total generation quantities for the binding and non-binding cases as  $x^B$  and  $x^{NB}$ , respectively. They are given by

$$x^B = \frac{(1-\delta)G + \delta(W+S)}{R[(1-\delta)^2 \alpha + 2\delta(1-\delta)\beta + \delta^2 \gamma]} \quad (3.28)$$

$$x^{NB} = \frac{G\gamma - (W+S)\beta + (W+S)\alpha - G\beta}{R(\alpha\gamma - \beta^2)} \quad (3.29)$$

Substituting the RPS binding solution of (3.21) and (3.22) into (3.10) and (3.12), we have the expected value and variance of  $NE_p$  for the binding case as follows,

$$E(NE_p)^B = \frac{[(1-\delta)G + \delta(W+S)]^2}{R[(1-\delta)^2 \alpha + 2\delta(1-\delta)\beta + \delta^2 \gamma]} \quad (3.30)$$



$$\text{var}(NE_p)^B = \frac{[(1-\delta)G + \delta(W+S)]^2}{R^2[(1-\delta)^2\alpha + 2\delta(1-\delta)\beta + \delta^2\gamma]} \quad (3.31)$$

Substituting in the RPS non-binding solution of (3.25) and (3.26) into (3.10) and (3.12) we have the expected value and variance of  $NE_p$  for the non-binding case as follows

$$E(NE_p)^{NB} = \frac{G^2\gamma - 2G(W+S)\beta + (W+S)^2\alpha}{R(\alpha\gamma - \beta^2)} \quad (3.32)$$

$$\text{var}(NE_p)^{NB} = \frac{G^2\gamma - 2G(W+S)\beta + (W+S)^2\alpha}{R^2(\alpha\gamma - \beta^2)} \quad (3.33)$$

The corresponding MV utility levels for the binding and non-binding cases are

$$U^B = \frac{[(1-\delta)G + \delta(W+S)]^2}{2R[(1-\delta)^2\alpha + 2\delta(1-\delta)\beta + \delta^2\gamma]} \quad (3.34)$$

$$U^{NB} = \frac{G^2\gamma - 2G(W+S)\beta + (W+S)^2\alpha}{2R(\alpha\gamma - \beta^2)} \quad (3.35)$$

## CHAPTER 4. PARAMETRIC ANALYSES

As mentioned before, local sensitivity analyses are inadequate to investigate the competing claims under a single framework. Hence, in this chapter, we will conduct a series of parametric analyses on the optimal generation quantities and their corresponding utility levels within the entire regions of our interest within the entire regions of our interest with respect to the key parameters of the RPS level, standard deviations of the PTC level and the conventional fuel price, and their correlation coefficient.

### 4.1 Investigation Steps

For each targeted parameter, the process of investigation is given as follows:

Step 0. (Initialization) Specify the optimal solution according to the range of the value of the targeted parameter  $x$ . For simplicity we denote that for a certain solution (binding or non-binding), the lower and upper bounds of the parameter is  $x_L$  and  $x_U$ . We note that  $x_L$  and  $x_U$  could be  $-\infty$  and  $\infty$  for some certain range where there's no lower or upper bound.

Differentiate the objective values in (3.21), (3.22), (3.25), (3.26), (3.28), (3.29), (3.34) and (3.35) with  $x$ . For simplicity we denote one certain derivative as  $\frac{\partial f}{\partial x}$ .

It is possible for  $\frac{\partial f}{\partial x}$  to have a either quadratic or linear or constant numerator. Since it can be proved the denominator will always be positive, we can determine the shape of  $\frac{\partial f}{\partial x}$  by examining its numerator denoting by  $f_n$ .

1. For the derivatives with quadratic numerator

Step 1.1 Solve for the roots on  $x$  which makes  $\frac{\partial f}{\partial x} = 0$  (i.e.,  $f_n = 0$ ). For simplicity we denote the roots as  $r_A$  (the one with smaller numerator) and  $r_B$ .

Step 1.2. Compute  $r_A - x_L$ ,  $r_A - x_U$ ,  $r_B - x_L$  and  $r_B - x_U$  and obtain their signs (i.e., negative or positive).

Step 1.3. Determine the shape of  $\frac{\partial f}{\partial x}$  by the results we get from the previous step.

2. For the derivatives with linear numerator

Step 2.1. Compute the values of  $f_n|_{x=x_L}$  and  $f_n|_{x=x_U}$ .

Step 2.2. Obtain the sign of the two values.

Step 2.3. Determine the shape of  $\frac{\partial f}{\partial x}$  by the results we get from the previous step.

3. For the derivatives with constant numerator

Step 3.1. Obtain the sign of  $f_n$ .

Step 3.2. Determine the shape of  $\frac{\partial f}{\partial x}$  by the results we get from the previous step.

In the rest of this chapter, the investigation for each of the critical parameters will be comprehensively conducted by following the steps we summarize above.

## 4.2 Analysis on the RPS Level $\delta$

We notice that, when  $x_c^B$  and  $x_w^B$  are given by (3.21) and (3.22), if  $(x_c^B, x_w^B)$  is the optimal solution, (3.24) must hold from Property 1. By rearranging (3.24) as a lower bound on  $\delta$  we have

$$\delta \geq \frac{(W+S)\alpha - G\beta}{(W+S)\alpha - G\beta + G\gamma - (W+S)\beta} \quad (4.1)$$

Similarly, when  $x_c^{NB}$  and  $x_w^{NB}$  are given by (3.25) and (3.26), if  $(x_c^{NB}, x_w^{NB})$  is the optimal solution, (3.27) must hold from Property 2 and can be rearranged as an upper bound on  $\delta$ . That is

$$\delta < \frac{(W+S)\alpha - G\beta}{(W+S)\alpha - G\beta + G\gamma - (W+S)\beta} \quad (4.2)$$

For simplification we denote  $\bar{\delta} = \frac{(W+S)\alpha - G\beta}{(W+S)\alpha - G\beta + G\gamma - (W+S)\beta}$ . The numerator and denominator of  $\bar{\delta}$  are both positive by (3.25) and (3.26), and the denominator must be greater than the numerator, thus we have  $0 < \bar{\delta} < 1$ .

Therefore,  $(x_c^B, x_w^B)$  is optimal when  $\bar{\delta} \leq \delta < 1$ , while  $(x_c^{NB}, x_w^{NB})$  is optimal when  $0 < \delta < \bar{\delta}$ , as showed in the following table.

**Table 3. Optimality conditions on  $\delta$**

| $\delta$  | Binding optimality condition   | Non-binding optimality condition |
|---|--------------------------------|----------------------------------|
|   | $\bar{\delta} \leq \delta < 1$ | $0 < \delta < \bar{\delta}$      |
| $\bar{\delta} = \frac{(W+S)\alpha - G\beta}{(W+S)\alpha - G\beta + G\gamma - (W+S)\beta}$ |                                |                                  |

Differentiating (3.21), (3.22), (3.25), (3.26), (3.28), (3.29), (3.34) and (3.35) with respect to  $\delta$ , we have

$$\frac{\partial x_c^B}{\partial \delta} = \frac{-2(1-\delta)^2 G\beta - 2\delta(1-\delta)G\gamma + (1-\delta)^2(W+S)\alpha - \delta^2(W+S)\gamma}{R[(1-\delta)^2\alpha + 2\delta(1-\delta)\beta + \delta^2\gamma]^2} \quad (4.3)$$

$$\frac{\partial x_w^B}{\partial \delta} = \frac{(1-\delta)^2 G\alpha - \delta^2 G\gamma + 2\delta(1-\delta)(W+S)\alpha + 2\delta^2(W+S)\beta}{R[(1-\delta)^2\alpha + 2\delta(1-\delta)\beta + \delta^2\gamma]^2} \quad (4.4)$$

$$\frac{\partial x_c^B}{\partial \delta} = \frac{\left( (1-\delta)^2 G\alpha - \delta(2-\delta)G\gamma + (1+\delta)(1-\delta)(W+S)\alpha \right) + 2\delta^2(W+S)\beta - 2(1-\delta)^2 G\beta - \delta^2(W+S)\gamma}{R[(1-\delta)^2\alpha + 2\delta(1-\delta)\beta + \delta^2\gamma]^2} \quad (4.5)$$

$$\frac{\partial U^B}{\partial \delta} = \frac{[(1-\delta)G + \delta(W+S)][-G((1-\delta)\beta + \delta\gamma) + (W+S)((1-\delta)\alpha + \delta\beta)]}{R[(1-\delta)^2\alpha + 2\delta(1-\delta)\beta + \delta^2\gamma]^2} \quad (4.6)$$

$$\frac{\partial x_c^{NB}}{\partial \delta} = \frac{\partial x_w^{NB}}{\partial \delta} = \frac{\partial x^{NB}}{\partial \delta} = \frac{\partial U^{NB}}{\partial \delta} = 0 \quad (4.7)$$

We note that when  $0 < \delta < \bar{\delta}$ , there is no change in the optimal generation quantities and corresponding utility level.

We further note that, when  $\delta = \bar{\delta}$ , there will be two distinctive slopes at that point of  $\bar{\delta}$ . In the direction of a decrease, the results will be the same as before. On the other hand, in the direction of an increase, the results will be the same as when  $\bar{\delta} < \delta < 1$ , which is summarized in the following proposition.

**Proposition 1** If  $\rho_{FS} > 0$ ,  $G > 0$ , and  $W + S > 0$  as implied by Fundamental Model Assumption 4, Technical Assumptions 2 and 3, then

- i)  $\frac{\partial U^B}{\partial \delta}$  is negative in  $(\bar{\delta}, 1)$
- ii)  $\frac{\partial x_c^B}{\partial \delta}$  is negative in  $(\bar{\delta}, 1)$
- iii)  $\frac{\partial x_w^B}{\partial \delta}$  is positive-negative in  $(\bar{\delta}, 1)$

iv)  $\frac{\partial x^B}{\partial \delta}$  is positive-negative in  $(\bar{\delta}, 1)$  when  $G - (W + S) > 0$ , and negative in  $(\bar{\delta}, 1)$

when  $G - (W + S) \leq 0$ .

See the Appendix for the proof. Statements i) and ii) indicate that when  $\delta$  exceeds the level of  $\bar{\delta}$ , the utility level and the generation quantity from the conventional source will decrease as  $\delta$  increases. i.e., after  $\bar{\delta}$ , any additional RPS requirement will be perceived negatively by the producer, and will affect the generation quantity from the conventional source negatively. This implies that one of the stated goals of RPS of “ensuring adequate supply of electric power” (see IL Public Act) may be jeopardized given a sufficiently high RPS level.

Statement iii) implies that a higher RPS level does not necessarily encourage more generation from the wind energy. Specifically, let us define  $\delta_w^*$  to be the RPS level that maximizes the generation quantity from the wind energy and  $\tilde{\delta}_w$  to be the RPS level from which the generation quantity from the wind energy starts to decrease relative to the case of no RPS ( $\delta_w^* < \tilde{\delta}_w$ ).

Mathematically, for  $\delta_w^*$  there exist two explicit expressions. Namely,

1) when  $G\alpha - G\gamma - 2(W + S)\alpha + 2(W + S)\beta \neq 0$

$$\delta_w^* = \frac{G\alpha - (W + S)\alpha - \sqrt{\alpha [G^2\gamma - 2G(W + S)\beta + (W + S)^2\alpha]}}{G\alpha - G\gamma - 2(W + S)\alpha + 2(W + S)\beta} \quad (4.8)$$

2) when  $G\alpha - G\gamma - 2(W + S)\alpha + 2(W + S)\beta = 0$

$$\delta_w^* = \frac{G}{2G - 2(W + S)} \quad (4.9)$$

We note that the difference between these two are arithmetic in nature. That is, 1) is from the quadratic form of the derivative and 2) is from the linear form. The general shape of  $x_w^B$  is the same for both cases.

Mathematically, on the other hand,  $\tilde{\delta}_w$  can be attained by solving  $x_w^B = x_w^{NB}$ , for as the larger of the two roots (the smaller root is  $\bar{\delta}$ ). Namely,

$$\tilde{\delta}_w = \frac{(W+S)\alpha^2 - G\alpha\beta - (W+S)\alpha\beta + G\alpha\gamma}{[(W+S)\alpha - G\beta + G\gamma - (W+S)\beta](\alpha - \beta)} \quad (4.10)$$

As  $\delta$  starts to exceed  $\bar{\delta}$ , the producer finds it optimal to increase generation from the wind energy and decrease generation from the conventional source to meet the increased RPS level. Once  $\delta$  exceeds  $\delta_w^*$ , however, the producer finds it optimal to reduce generation from both sources as the more stringent required RPS further reduces his/her ability to maximize his/her utility by reducing the size of the region for feasible solutions. As  $\delta$  starts to exceed  $\tilde{\delta}_w$ , the regulatory condition for generation business deteriorates such a degree that the generation quantity from the wind energy is even less than that without the RPS.

Statement iv) indicates there are two possible shapes of  $\frac{\partial x^B}{\partial \delta}$  in  $(\bar{\delta}, 1)$  which depend on whether  $G - (W + S) > 0$  (i.e., the expected net earnings after tax per MWh from the conventional source are greater than the expected modified net earnings per MWh from the wind energy plus the expected benefit from the PTC per MWh) or  $G - (W + S) \leq 0$ .

The cost of generation from the wind energy varies greatly due to numerous factors such as the size of a wind farm, the method of financing *etc.* (Johnson, 2009). For example, in Lazard's levelized cost of energy analysis (2009), the levelized generation cost from the

wind energy (reflecting the PTC) varies from \$57 per MWh to \$113 per MWh, while the generation cost from some conventional source (specifically for a natural-gas combined cycle generator) varies from \$74 per MWh to \$102 per MWh. Hence, a priori, it is unclear whether  $G - (W + S) > 0$  or not. Therefore, it is necessary to consider both cases.

When  $G - (W + S) > 0$ , the total generation quantity will first increase then decrease. For such a case, we denote  $\delta^*$  as the  $\delta$  level maximizing the total generation quantity,  $\tilde{\delta}$  as the  $\delta$  level from which the total generation quantity starts to decrease relative to the case of no RPS ( $\delta^* < \tilde{\delta}$ ).

From mathematical steps analogous to those in the analysis of statement iii) we have

$$\delta^* = \frac{G(\alpha + \gamma - 2\beta) - \sqrt{(\alpha + \gamma - 2\beta)[G^2\gamma - 2G(W + S)\beta + (W + S)^2\alpha]}}{[G - (W + S)](\alpha + \gamma - 2\beta)} \quad (4.11)$$

$$\tilde{\delta} = \frac{G\alpha\gamma - G\beta\gamma - 2(W + S)\alpha\beta + (W + S)\beta^2 + (W + S)\alpha^2 - G\alpha\beta + G\beta^2}{[(W + S)\alpha - G\beta + G\gamma - (W + S)\beta](\alpha - 2\beta + \gamma)} \quad (4.12)$$

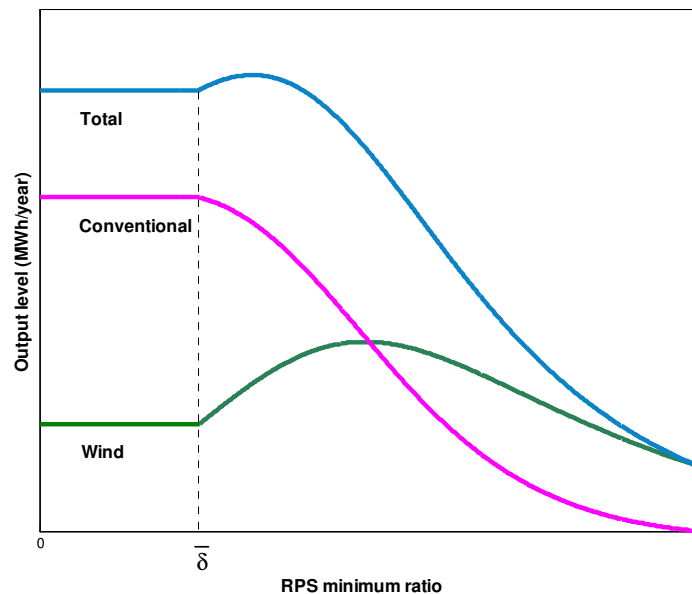
We note that both  $\delta^*$  and  $\tilde{\delta}$  can be shown to be smaller than 1. For the total generation quantity, the slope of  $x^B$  is positive in  $(\tilde{\delta}, \delta^*)$  and negative in  $(\delta^*, 1)$ , and it can be further shown that  $\tilde{\delta} < \tilde{\delta}_w$  which indicates that there exists an effective range of RPS,  $(\tilde{\delta}, \tilde{\delta}_w)$ , in which the generation quantity from the wind energy as well as the total generation quantity both increase. Hence, if the level of  $\delta$  is set by the policy maker within such a range, such a  $\delta$  level will be consistent with several critical stated goals of the RPS such as “to increase the reliance on renewable energy” (see CA Public Utilities Code), “to ensure adequate energy supply” (see IL Public Act), and “to reduce the reliance on imported fuels”



(see CT Docket). This will be an important observation for the policy implications, which will be presented in Chapter 6 later.

When  $G - (W + S) \leq 0$ , the total generation quantity will always decrease as  $\delta$  increases from  $\bar{\delta}$ , comparing to the level of non-binding optimal solution. Hence, for example, if the expected PTC level is sufficiently high, the RPS goal of ensuring adequate supply of electric power may be unattainable. This insight may serve as a warning to any state implementing the RPS “mechanically” based on “conventional wisdom”.

Figure 3 and Figure 4 below illustrate the impact of  $\delta$  parametrically on the generation portfolio when  $G - (W + S) > 0$  and  $G - (W + S) \leq 0$ , respectively.



**Figure 3. Impact of  $\delta$  on optimal generation portfolio when  $G - (W + S) > 0$**

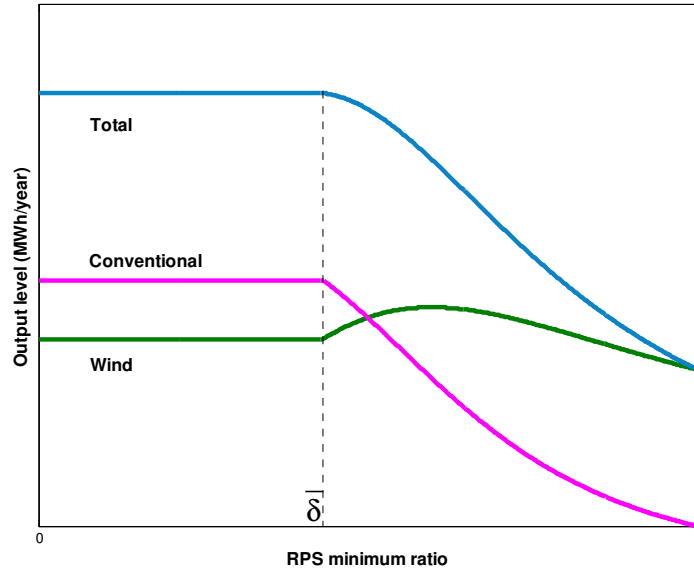


Figure 4. Impact of  $\delta$  on optimal generation portfolio when  $G - (W + S) \leq 0$

### 4.3 Analysis on the Standard Deviation $\sigma_s$

Let us now examine the analysis on  $\sigma_s$ , the standard deviation of the PTC level. Since  $\beta = -(1-\tau)h\sigma_F\sigma_s\rho_{FS}$  by definition, let  $\beta = \beta_1\sigma_s$  where  $\beta_1 = -(1-\tau)h\sigma_F\rho_{FS} < 0$ . Also, we note  $\gamma = \sigma_s^2$  by definition. Substituting the right hand sides of  $\beta$  and  $\gamma$  into conditions (3.24), (3.27) and rearranging them leads us to a lower bound and an upper bound on  $\sigma_s$  that are analogous to the ones derived for  $\delta$  in the previous subsection.

Hence,  $(x_c^B, x_w^B)$  is optimal when  $\sigma_s \geq \bar{\sigma}_s$ , while  $(x_c^{NB}, x_w^{NB})$  is optimal when  $0 < \sigma_s < \bar{\sigma}_s$ , where

$$\bar{\sigma}_s = \frac{-[(1-\delta)G - \delta(W+S)]\beta_1 + \sqrt{[(1-\delta)G - \delta(W+S)]^2\beta_1^2 + 4\delta(1-\delta)G(W+S)\alpha}}{2\delta G}$$

The optimality conditions are summarized in the following table.

**Table 4. Optimality conditions on  $\sigma_s$** 

| $\sigma_s$         | Binding optimality solution  | Non-binding optimality condition |
|--------------------|--|----------------------------------|
|                    | $\sigma_s \geq \bar{\sigma}_s$   | $0 < \sigma_s < \bar{\sigma}_s$  |
| $\bar{\sigma}_s =$ | $\frac{-[(1-\delta)G - \delta(W+S)]\beta_1 + \sqrt{[(1-\delta)G - \delta(W+S)]^2 \beta_1^2 + 4\delta(1-\delta)G(W+S)\alpha}}{2\delta G}$ |                                  |

Differentiating (3.21), (3.22), (3.25), (3.26), (3.28), (3.29), (3.34) and (3.35) with respect to  $\sigma_s$ , we have

$$\frac{\partial x_c^B}{\partial \sigma_s} = \frac{-2[(1-\delta)^2 G + \delta(1-\delta)(W+S)][\delta(1-\delta)\beta_1 + \delta^2 \sigma_s]}{R[(1-\delta)^2 \alpha + 2\delta(1-\delta)\beta_1 \sigma_s + \delta^2 \sigma_s^2]^2} \quad (4.13)$$

$$\frac{\partial x_w^B}{\partial \sigma_s} = \frac{-2[\delta(1-\delta)G + \delta^2(W+S)][\delta(1-\delta)\beta_1 + \delta^2 \sigma_s]}{R[(1-\delta)^2 \alpha + 2\delta(1-\delta)\beta_1 \sigma_s + \delta^2 \sigma_s^2]^2} \quad (4.14)$$

$$\frac{\partial x^B}{\partial \sigma_s} = \frac{-2[(1-\delta)G + \delta(W+S)][\delta(1-\delta)\beta_1 + \delta^2 \sigma_s]}{R[(1-\delta)^2 \alpha + 2\delta(1-\delta)\beta_1 \sigma_s + \delta^2 \sigma_s^2]^2} \quad (4.15)$$

$$\frac{\partial U^B}{\partial \sigma_s} = \frac{-[(1-\delta)G + \delta(W+S)]^2 [\delta(1-\delta)\beta_1 + \delta^2 \sigma_s]}{R[(1-\delta)^2 \alpha + 2\delta(1-\delta)\beta_1 \sigma_s + \delta^2 \sigma_s^2]^2} \quad (4.16)$$

$$\frac{\partial x_c^{NB}}{\partial \sigma_s} = \frac{(W+S)\beta_1}{R(\alpha - \beta_1^2)\sigma_s^2} \quad (4.17)$$

$$\frac{\partial x_w^{NB}}{\partial \sigma_s} = \frac{G\beta_1 \sigma_s - 2(W+S)\alpha}{R(\alpha - \beta_1^2)\sigma_s^3} \quad (4.18)$$

$$\frac{\partial x^{NB}}{\partial \sigma_s} = \frac{(W+S)\beta_1 + G\beta_1 \sigma_s - 2(W+S)\alpha}{R(\alpha - \beta_1^2)\sigma_s^3} \quad (4.19)$$

$$\frac{\partial U^{NB}}{\partial \sigma_s} = \frac{G(W+S)\beta_1\sigma_s - (W+S)^2\alpha}{R(\alpha - \beta_1^2)\sigma_s^3} \quad (4.20)$$

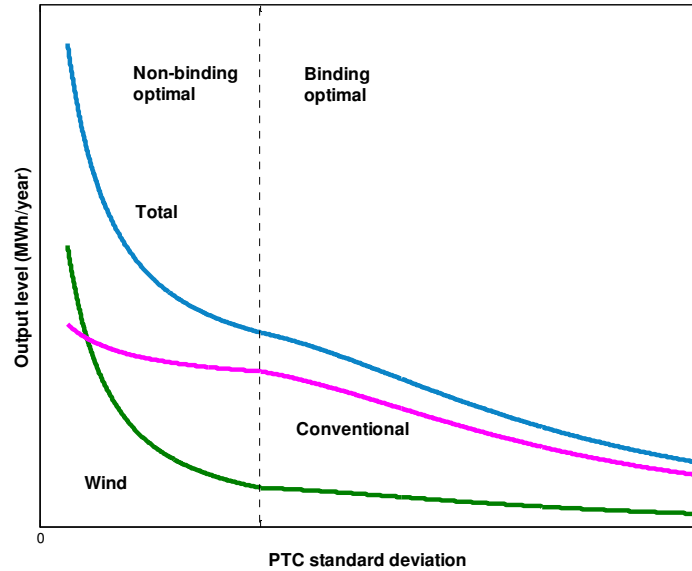
Since  $\beta_1 < 0$ , it can be easily seen that  $\frac{\partial x_c^{NB}}{\partial \sigma_s}$ ,  $\frac{\partial x_w^{NB}}{\partial \sigma_s}$ ,  $\frac{\partial x^{NB}}{\partial \sigma_s}$ , and  $\frac{\partial U^{NB}}{\partial \sigma_s}$  are all negative. This indicates that when  $0 < \sigma_s < \bar{\sigma}_s$ , all the generation quantities (total, from the conventional source and from the wind energy) as well as the producer's utility will decrease as  $\sigma_s$  increases. This is consistent with the intuitive insight that uncertainty in the PTC level will discourage the wind generation. In addition, the same uncertainty, given everything else remains the same, will also discourage the generation from the conventional source, and reduces the producer's utility at optimality.

We further note that, when  $\sigma_s = \bar{\sigma}_s$ , there will be two distinctive slopes at that point of  $\bar{\sigma}_s$ . In the direction of a decrease, the results will be the same as before. On the other hand, in the direction of an increase, the results will be the same as when  $\sigma_s > \bar{\sigma}_s$ , which is summarized in the following proposition.

**Proposition 2** If  $\rho_{FS} > 0$ ,  $G > 0$ , and  $W + S > 0$  as implied by Fundamental Model Assumption 4, Technical Assumptions 2 and 3,  $\frac{\partial x_c^B}{\partial \sigma_s}$ ,  $\frac{\partial x_w^B}{\partial \sigma_s}$ ,  $\frac{\partial x^B}{\partial \sigma_s}$ , and  $\frac{\partial U^B}{\partial \sigma_s}$  are negative in when  $\sigma_s > \bar{\sigma}_s$ .

See Appendix for the proof. With Proposition 2, we note that, whether the optimal solution is binding or not, greater uncertainty in the PTC level will always discourage not only the generation quantity from the wind energy, but also that from the conventional source

as well as the producer's utility. Figure 5 below illustrates the impact of  $\sigma_s$  parametrically on the generation portfolio.



**Figure 5. Impact of  $\sigma_s$  on optimal generation portfolio**

We further note that, When  $\sigma_s$  is sufficiently small ( $\approx 0$ ), it can be verified from the non-binding optimal solution of (3.25) and (3.26) that  $x_c^{NB}|_{\sigma_s=0} = 0 < x_w^{NB}|_{\sigma_s=0}$ . This implies that the producer will prefer generation from the wind energy to that from the conventional source. As  $\sigma_s$  increases from  $\approx 0$ , the producer finds it optimal to reduce the generation quantities from both sources as well as the fraction from the wind energy. Once  $\sigma_s$  exceeds  $\bar{\sigma}_s$ , the producer will continue to reduce generation from both sources to maximize his/her utility with the fraction from the wind energy equaling to the required minimum RPS level.

#### 4.4 Analysis on the Standard Deviation $\sigma_F$

Now we proceed to examine the analysis on  $\sigma_F$ , the standard deviation of the conventional fuel price. With an approach that is similar to the one used in the analysis on  $\sigma_S$ , we have  $\alpha = \alpha_1 \sigma_F^2$ ,  $\beta = \beta_2 \sigma_F$  where  $\alpha_1 = (1-\tau)^2 h^2 > 0$  and  $\beta_2 = -(1-\tau)h\sigma_S \rho_{FS} < 0$ . Substituting the right hand sides of  $\alpha$  and  $\beta$  into conditions (3.24), (3.27) and rearranging them leads us to an upper bound and a lower bound on  $\sigma_F$ .

Hence,  $(x_c^B, x_w^B)$  is optimal when  $0 < \sigma_F \leq \bar{\sigma}_F$ , while  $(x_c^{NB}, x_w^{NB})$  is optimal when  $\sigma_F > \bar{\sigma}_F$ , where

$$\bar{\sigma}_F = \frac{[(1-\delta)G - \delta(W+S)]\beta_2 + \sqrt{[(1-\delta)G - \delta(W+S)]^2 \beta_2^2 + 4\delta(1-\delta)G(W+S)\alpha_1\gamma}}{2(1-\delta)(W+S)\alpha_1}$$

Table 5 below summarizes the optimality conditions.

**Table 5. Optimality conditions on  $\sigma_F$**

| $\sigma_F$ | Binding optimality condition  | Non-binding optimality condition |
|------------|---|----------------------------------|
|            | $0 < \sigma_F \leq \bar{\sigma}_F$  | $\sigma_F > \bar{\sigma}_F$      |
|            | $\bar{\sigma}_F = \frac{[(1-\delta)G - \delta(W+S)]\beta_2 + \sqrt{[(1-\delta)G - \delta(W+S)]^2 \beta_2^2 + 4\delta(1-\delta)G(W+S)\alpha_1\gamma}}{2(1-\delta)(W+S)\alpha_1}$ |                                  |

Differentiating (3.21), (3.22), (3.25), (3.26), (3.28), (3.29), (3.34) and (3.35) with respect to  $\sigma_F$ , we have

$$\frac{\partial x_c^B}{\partial \sigma_F} = \frac{-2[(1-\delta)^2 G + \delta(1-\delta)(W+S)][(1-\delta)^2 \alpha_1 \sigma_F + \delta(1-\delta)\beta_2]}{R[(1-\delta)^2 \alpha_1 \sigma_F^2 + 2\delta(1-\delta)\beta_2 \sigma_F + \delta^2 \gamma]^2} \quad (4.21)$$

$$\frac{\partial x_w^B}{\partial \sigma_F} = \frac{-2[\delta(1-\delta)G + \delta^2(W+S)][(1-\delta)^2\alpha_1\sigma_F + \delta(1-\delta)\beta_2]}{R[(1-\delta)^2\alpha_1\sigma_F^2 + 2\delta(1-\delta)\beta_2\sigma_F + \delta^2\gamma]^2} \quad (4.22)$$

$$\frac{\partial x^B}{\partial \sigma_F} = \frac{-2[(1-\delta)G + \delta(W+S)][(1-\delta)^2\alpha_1\sigma_F + \delta(1-\delta)\beta_2]}{R[(1-\delta)^2\alpha_1\sigma_F^2 + 2\delta(1-\delta)\beta_2\sigma_F + \delta^2\gamma]^2} \quad (4.23)$$

$$\frac{\partial U^B}{\partial \sigma_F} = \frac{-[(1-\delta)G + \delta(W+S)]^2[(1-\delta)^2\alpha_1\sigma_F + \delta(1-\delta)\beta_2]}{R[(1-\delta)^2\alpha_1\sigma_F^2 + 2\delta(1-\delta)\beta_2\sigma_F + \delta^2\gamma]^2} \quad (4.24)$$

$$\frac{\partial x_c^{NB}}{\partial \sigma_F} = \frac{(W+S)\beta_2\sigma_F - 2G\gamma}{R(\alpha_1\gamma - \beta_2^2)\sigma_F^3} \quad (4.25)$$

$$\frac{\partial x_w^{NB}}{\partial \sigma_F} = \frac{G\beta_2}{R(\alpha_1\gamma - \beta_2^2)\sigma_F^2} \quad (4.26)$$

$$\frac{\partial x^{NB}}{\partial \sigma_F} = \frac{(W+S)\beta_2\sigma_F - 2G\gamma + G\beta_2}{R(\alpha_1\gamma - \beta_2^2)\sigma_F^2} \quad (4.27)$$

$$\frac{\partial U^{NB}}{\partial \sigma_F} = \frac{G(W+S)\beta_2\sigma_F - G^2\gamma}{R(\alpha_1\gamma - \beta_2^2)\sigma_F^3} \quad (4.28)$$

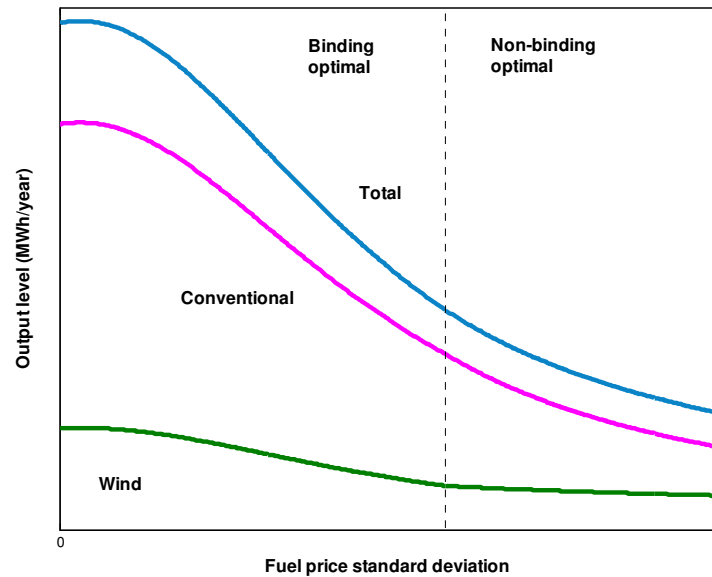
Since  $\beta_2 < 0$ , it is easy to tell  $\frac{\partial x_c^{NB}}{\partial \sigma_F}$ ,  $\frac{\partial x_w^{NB}}{\partial \sigma_F}$ ,  $\frac{\partial x^{NB}}{\partial \sigma_F}$ , and  $\frac{\partial U^{NB}}{\partial \sigma_F}$  are all negative. This

implies that when P1 is non-binding optimal (i.e.,  $\sigma_F$  is sufficiently large that  $\sigma_F > \bar{\sigma}_F$ ), the generation quantities from both sources, as well as the producer's utility, will decrease as  $\sigma_F$  increases.

We further note that, when  $\sigma_F = \bar{\sigma}_F$ , there will be two distinctive slopes at that point of  $\bar{\sigma}_F$ . In the direction of an increase, the results will be the same as before. On the other hand, in the direction of a decrease, the results will be the same as when  $0 < \sigma_F < \bar{\sigma}_F$ , which is summarized in the following proposition.

**Proposition 3** If  $\rho_{FS} > 0$ ,  $G > 0$ , and  $W + S > 0$  as implied by Fundamental Model Assumption 4, Technical Assumptions 2 and 3,  $\frac{\partial x_c^B}{\partial \sigma_F}$ ,  $\frac{\partial x_w^B}{\partial \sigma_F}$ ,  $\frac{\partial x^B}{\partial \sigma_F}$ , and  $\frac{\partial U^B}{\partial \sigma_F}$  are positive-negative when  $0 < \sigma_F < \bar{\sigma}_F$ .

See Appendix for the proof. From Proposition 3, we observe that increasing uncertainty in the conventional fuel price does not always discourage generation when it is positively correlated with the wind power PTC. Instead, when  $\sigma_F$  is sufficiently small, an increase in  $\sigma_F$  will actually encourage generation from both sources as well as increase the producer's utility. Figure 6 below illustrates the impact of  $\sigma_F$  parametrically on the generation portfolio.



**Figure 6. Impact of  $\sigma_F$  on optimal generation portfolio**



The observation from Proposition 3 contradicts a preliminary intuition that the uncertainty in fuel price is a disincentive to electric power generation. Hence, it deserves further scrutiny as follows.

Let us denote  $\sigma_F^*$  as the  $\sigma_F$  level when the generation quantities from both sources are maximized. As it can be obtained from Appendix,  $\sigma_F^* = \frac{-\delta\beta_2}{(1-\delta)\alpha_1}$ . When  $\sigma_F$  increases from some level between 0 and  $\sigma_F^*$ , the producer finds it optimal to increase both generation quantities from both sources. Mathematically, from  $\alpha = (1-\tau)^2 h^2 \sigma_F^2$  in Chapter 3, an increase in  $\sigma_F$  contributes to an increase in  $\text{var}(NE_p) = \alpha x_c^2 + 2\beta x_c x_w + \gamma x_w^2$ . At the same time, from  $\beta = -(1-\tau)h\sigma_F\sigma_S\rho_{FS}$ , an increase in  $\sigma_F$  contributes to a decrease in  $\text{var}(NE_p)$ . Due to these two concurrent effects plus an increase anticipated in the expected net earnings of the portfolio, the producer may find the increase in  $\sigma_F$  as an incentive to produce more from both sources when  $\sigma_F$  is sufficiently small.

#### 4.5 Analysis on the Correlation Coefficient $\rho_{FS}$

In this section we examine the analysis on  $\rho_{FS}$ , which represents the degree to which the movements of the conventional fuel price and the PTC level are associated. Similarly to those analyses on  $\sigma_S$  and  $\sigma_F$ , we have  $\beta = \beta_3\rho_{FS}$  by definition where  $\beta_3 = -(1-\tau)h\sigma_F\sigma_S < 0$ . By substituting the right hand sides of  $\beta$  into conditions (3.24) and (3.27) and rearranging them, we can obtain the four explicit sets of optimality conditions as below. Namely,

- 1) If  $(1-\delta)G - \delta(W+S) < 0$  and  $0 < \bar{\rho}_{FS} < 1$ ,

$(x_c^B, x_w^B)$  is optimal when  $\bar{\rho}_{FS} \leq \rho_{FS} < 1$ , while  $(x_c^{NB}, x_w^{NB})$  is optimal when  $0 < \rho_{FS} < \bar{\rho}_{FS}$ .

2) If  $(1-\delta)G - \delta(W+S) > 0$  and  $0 < \bar{\rho}_{FS} < 1$ ,

$(x_c^B, x_w^B)$  is optimal when  $0 < \rho_{FS} \leq \bar{\rho}_{FS}$ , while  $(x_c^{NB}, x_w^{NB})$  is optimal when  $\bar{\rho}_{FS} < \rho_{FS} < 1$ .

3) If  $(1-\delta)G - \delta(W+S) = 0$  and  $(1-\delta)(W+S)\alpha - \delta G\gamma \leq 0$ ,

or  $(1-\delta)G - \delta(W+S) < 0$  and  $\bar{\rho}_{FS} \leq 0$ ,

or  $(1-\delta)G - \delta(W+S) > 0$  and  $\bar{\rho}_{FS} \geq 1$ ,

$(x_c^B, x_w^B)$  is optimal for any  $0 < \rho_{FS} < 1$ .

4) If  $(1-\delta)G - \delta(W+S) = 0$  and  $(1-\delta)(W+S)\alpha - \delta G\gamma > 0$ ,

or  $(1-\delta)G - \delta(W+S) < 0$  and  $\bar{\rho}_{FS} \geq 1$ ,

or  $(1-\delta)G - \delta(W+S) > 0$  and  $\bar{\rho}_{FS} \leq 0$ ,

$(x_c^{NB}, x_w^{NB})$  is optimal for any  $0 < \rho_{FS} < 1$ .

where  $\bar{\rho}_{FS} = \frac{(1-\delta)(W+S)\alpha - \delta G\gamma}{[(1-\delta)G - \delta(W+S)]\beta_3}$  for all the conditions above. The four cases above are

summarized in Table 6 below.

**Table 6. Optimality conditions on  $\rho_{FS}$**

| $\rho_{FS}$  | Binding optimality condition         | Non-binding optimality condition  |
|--|--------------------------------------|-----------------------------------|
| $(1-\delta)G - \delta(W+S) < 0$<br>$0 < \bar{\rho}_{FS} < 1$ | $\bar{\rho}_{FS} \leq \rho_{FS} < 1$ | $0 < \rho_{FS} < \bar{\rho}_{FS}$ |

|   |                                      |                                   |
|---|--------------------------------------|-----------------------------------|
| $(1-\delta)G - \delta(W+S) > 0$<br>$0 < \bar{\rho}_{FS} < 1$  | $0 < \rho_{FS} \leq \bar{\rho}_{FS}$ | $\bar{\rho}_{FS} < \rho_{FS} < 1$ |
| $(1-\delta)G - \delta(W+S) = 0,$<br>$(1-\delta)(W+S)\alpha - \delta G\gamma \leq 0$<br>or $(1-\delta)G - \delta(W+S) < 0,$<br>$\bar{\rho}_{FS} \leq 0$<br>or $(1-\delta)G - \delta(W+S) > 0,$<br>$\bar{\rho}_{FS} \geq 1$ | any $0 < \rho_{FS} < 1$              | /                                 |
| $(1-\delta)G - \delta(W+S) = 0,$<br>$(1-\delta)(W+S)\alpha - \delta G\gamma > 0$<br>or $(1-\delta)G - \delta(W+S) < 0,$<br>$\bar{\rho}_{FS} \geq 1$<br>or $(1-\delta)G - \delta(W+S) > 0,$<br>$\bar{\rho}_{FS} \leq 0$    | /                                    | any $0 < \rho_{FS} < 1$           |
| $\bar{\rho}_{FS} = \frac{(1-\delta)(W+S)\alpha - \delta G\gamma}{[(1-\delta)G - \delta(W+S)]\beta_3}$   |                                      |                                   |

Substituting  $\beta$  with  $\beta_3\rho_{FS}$  into (3.21), (3.22), (3.25), (3.26), (3.28), (3.29), (3.34)

and (3.35) and differentiating them with respect to  $\rho_{FS}$ , we have

$$\frac{\partial x_c^B}{\partial \rho_{FS}} = \frac{-2\delta(1-\delta)[(1-\delta)^2G + \delta(1-\delta)(W+S)]\beta_3}{R[(1-\delta)^2\alpha + 2\delta(1-\delta)\beta_3\rho_{FS} + \delta^2\gamma]^2} \quad (4.29)$$

$$\frac{\partial x_w^B}{\partial \rho_{FS}} = \frac{-2\delta(1-\delta)[\delta(1-\delta)G + \delta^2(W+S)]\beta_3}{R[(1-\delta)^2\alpha + 2\delta(1-\delta)\beta_3\rho_{FS} + \delta^2\gamma]^2} \quad (4.30)$$

$$\frac{\partial x^B}{\partial \rho_{FS}} = \frac{-2\delta(1-\delta)[(1-\delta)G + \delta(W+S)]\beta_3}{R[(1-\delta)^2\alpha + 2\delta(1-\delta)\beta_3\rho_{FS} + \delta^2\gamma]^2} \quad (4.31)$$

$$\frac{\partial U^B}{\partial \rho_{FS}} = \frac{-\delta(1-\delta)[(1-\delta)G + \delta(W+S)]^2\beta_3}{R[(1-\delta)^2\alpha + 2\delta(1-\delta)\beta_3\rho_{FS} + \delta^2\gamma]^2} \quad (4.32)$$

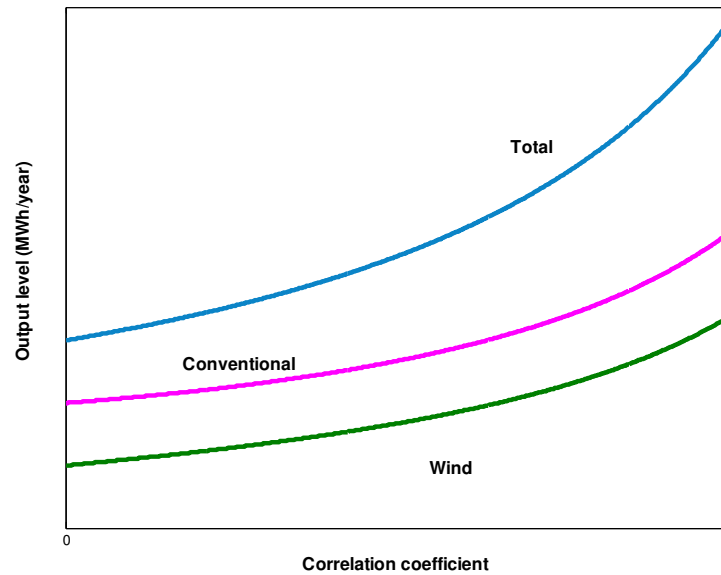
$$\frac{\partial x_c^{NB}}{\partial \rho_{FS}} = \frac{-(W+S)\beta_3(\alpha\gamma + \beta_3^2\rho_{FS}^2) + 2G\gamma\beta_3^2\rho_{FS}}{R(\alpha\gamma - \beta_3^2\rho_{FS}^2)^2} \quad (4.33)$$

$$\frac{\partial x_w^{NB}}{\partial \rho_{FS}} = \frac{-G\beta_3(\alpha\gamma + \beta_3^2\rho_{FS}^2) + 2(W+S)\alpha\beta_3^2\rho_{FS}}{R(\alpha\gamma - \beta_3^2\rho_{FS}^2)^2} \quad (4.34)$$

$$\frac{\partial x^{NB}}{\partial \rho_{FS}} = \frac{-[G + (W+S)]\beta_3(\alpha\gamma + \beta_3^2\rho_{FS}^2) + 2[G\gamma + (W+S)\alpha]\beta_3^2\rho_{FS}}{R(\alpha\gamma - \beta_3^2\rho_{FS}^2)^2} \quad (4.35)$$

$$\frac{\partial U^{NB}}{\partial \rho_{FS}} = \frac{-G(W+S)\beta_3R(\alpha\gamma - \beta_3^2\rho_{FS}^2) + [G^2\gamma + (W+S)^2\alpha]\beta_3^2\rho_{FS}}{R(\alpha\gamma - \beta_3^2\rho_{FS}^2)^2} \quad (4.36)$$

Since  $\beta_3 < 0$ , it can be easily see that all the derivatives are positive, implying the generation quantities from both sources will increase as  $\rho_{FS}$  increases, so does the portfolio's utility. Recall the formulation of the variance of  $NE_p$  in (3.11) as  $\text{var}(NE_p) = (1-\tau)^2 h^2 \sigma_F^2 x_c^2 - 2(1-\tau)h\sigma_F\sigma_S\rho_{FS}x_c x_w + \sigma_S^2 x_w^2$ , an increase in  $\rho_{FS}$  structurally reduces the variance when  $\rho_{FS} > 0$ . This implies that a higher level of correlation between the conventional fuel price and the PTC level is preferred by the risk-averse producer, and encourages generation from the wind energy. Figure 7 below illustrate the impact of  $\rho_{FS}$  parametrically on the generation portfolio.



**Figure 7. Impact of  $\rho_{FS}$  on optimal generation portfolio**

For the purpose of encouraging more generation from the wind energy, the current policy practiced quantitatively and qualitatively (Fundamental Model Assumption 4) seems to make sense. We further note that our analysis here not only provides some quantitative support for such a policy, but also shows the exact magnitudes of changes if the policy changes with respect to  $\rho_{FS}$ .

## CHAPTER 5. NUMERICAL EXAMPLE

In this chapter, we provide a numerical example to illustrate the key features of our model based on a mixture of empirical and hypothetical data. We recognize the accurate characterization of uncertainties is a challenging research problem in itself due to different durations of the PTC and special features such as retroactive effective durations (Wiser, 2007). We also notice that there exist various approaches that can be used to estimate the covariance and correlation coefficient between two non-synchronous processes (see Hayashi and Yoshida, 2005). Here we assume that the producer's conventional fuel source is natural gas, and the available data on the natural gas prices and the PTC levels are for the one month average values of January from 2002 to 2010 (e.g., one could assume that the producer's decisions are to be made in January of 2011).

**Table 7. January average natural gas price and PTC, 2002-2010**

| Year | Natural gas price for<br>January<br>(\$/Mbtu) | PTC<br>for January<br>(\$/MWh) |
|------|---|--------------------------------|
| 2002 | 3.1   | 0                              |
| 2003 | 5.33  | 18                             |
| 2004 | 6.37  | 0                              |
| 2005 | 6.72  | 19                             |
| 2006 | 9.15  | 19                             |
| 2007 | 7.08  | 20                             |
| 2008 | 8.52  | 21                             |
| 2009 | 6.59  | 21                             |
| 2010 | 6.97  | 22                             |

The monthly average natural gas prices for power sectors in Table 7 are obtained from EIA's summary (EIA, 2010b). For the PTC values, incomplete records could be found in some references such as \$1.8/MWh in 2003 from EIA (2005) and \$2.1/MWh in 2009 from Union of Concerned Scientists (2009). The comprehensive data we use are obtained from a combination of these published sources and some extrapolations.

Based on the data in Table 7, expected values, standard deviations, and correlation coefficients of the natural gas price and PTC level can be calculated in a straightforward manner (Everitt, 2006). The unbiased estimates of standard deviation ( $s_x$ ) and correlation

coefficient ( $r_{xy}$ ) are given by  $s_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$  and  $r_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}$ ,

respectively.

Table 8 shows the calculated results for both natural gas price and PTC. It can be seen that PTC and natural gas price are positively correlated, which is consistent with our justification in Chapter 3; the variance of the PTC is much greater than the variance of the natural gas price.

**Table 8. Expected values, standard deviations and correlation coefficient**

| Random variable             | Expected value | Standard deviation | Correlation coefficient |
|-----------------------------|----------------|--------------------|-------------------------|
| Natural gas price (\$/Mbtu) | 6.648          | 1.749              | $\rho_{FS}$ 0.634       |
| PTC (\$/MWh)                | 15.556         | 8.904              |                         |

For the remaining parameter values except the risk aversion factor, Table 9 below shows the hypothetical data such as the corporate income tax rate, maintenance and capital

costs, capacity factors and annualized capital costs. All the values are estimated or modified within a reasonable range from empirical data in EIA's reports such as Annual Energy Outlook 2010 (2010) and Electric Power Industry 2009 (2010).

**Table 9. Data for the remaining parameters except risk aversion factor**

| Descriptions  | Values            |
|---|-------------------|
| Corporate income tax rate ( $\tau$ )                                    | 35%               |
| Heat rate of the gas-fired power plant ( $h$ )                          | 7 Mbtu/MWh        |
| Maintenance cost of the gas-fired power plant ( $m_c$ )                 | 3 \$/MWh          |
| Maintenance cost of the wind farm ( $m_w$ )                             | 2 \$/MWh          |
| Capacity factor of the gas-fired power plant ( $k_c$ )                  | 40%               |
| Capacity factor of the wind farm ( $k_w$ )                              | 25%               |
| Annualized capital cost of the gas-fired power plant ( $d_c$ )          | 5%                |
| Annualized capital cost of the wind farm ( $d_w$ )                      | 10%               |
| Cost of installing unit capacity of the gas-fired power plant ( $c_c$ ) | 900,000 (\$/MW)   |
| Cost of installing unit capacity of the wind farm ( $c_w$ )             | 1,986,000 (\$/MW) |
| Number of hours per year ( $t$ )  | 8,760 (hours)     |
| Electricity selling price ( $p$ )                                       | 80 (\$/MWh)       |

Based on the data from we have

$$\alpha = (1 - \tau)^2 h^2 \sigma_F^2 = 63.329$$

$$\beta = -(1 - \tau) h \sigma_F \sigma_S \rho_{FS} = -44.924$$

$$\gamma = \sigma_S^2 = 79.281$$

We also obtained the expected net earnings per MWh after tax from the gas-fired power plant and the expected modified net earnings per MWh from the wind farm to be

$$G = 11.44 (\$/MWh)$$



$$W = -8.25 (\$/MWh)$$

Hence we have  $W + S = 7.31 (\$/MWh)$ . We note that in this case,  $G - (W + S) > 0$ .

Finally, we assume the risk aversion factor  $R = 10^{-7}$ .

## 5.1 Numerical Solution

In this subsection we suppose the RPS minimum ratio  $\delta = 0.2$ , i.e., at least 20% of the producer's generation quantity should come from the wind farm.

By applying the data to check the optimality conditions (3.24) and (3.27), we find that

$$\delta[G\gamma - (W + S)\beta] - (1 - \delta)[(W + S)\alpha - G\beta] = -534.741 < 0$$

Therefore in this case the non-binding solution is optimal, which can be obtained from (3.25) and (3.26) as

$$x_c = x_c^{NB} = \frac{G\gamma - (W + S)\beta}{R(\alpha\gamma - \beta^2)} = 4.118 \times 10^6 \text{ MWh/year}$$

$$x_w = x_w^{NB} = \frac{(W + S)\alpha - G\beta}{R(\alpha\gamma - \beta^2)} = 3.256 \times 10^6 \text{ MWh/year}$$

From these results, the total generation quantity is  $x^{NB} = 7.374 \times 10^6$  MWh/year and the corresponding producer's utility level is  $3.549 \times 10^7$ . We note that the fraction of generation from the wind energy is 0.442. Furthermore, it can be verified that the optimal capacity of the gas-fired power plant is 1175.228 MW while that of the wind farm is 1486.758 MW. In this case we observe that a RPS level of 20% is not effective to this producer since it doesn't affect his/her generation portfolio relative to the case of no RPS.

## 5.2 Impact of the RPS Level $\delta$

In this subsection we illustrate the significant properties of  $\delta$  according to our analysis in the previous section. We have

$$\bar{\delta} = \frac{(W + S)\alpha - G\beta}{(W + S)\alpha - G\beta + G\gamma - (W + S)\beta} = 0.442$$

Hence, the non-binding solution is optimal when  $0 < \delta < 0.442$  and binding solution is optimal when  $\delta \geq 0.442$ . It can be verified that if  $\delta = 0.45$ , the optimal solution is

$$x_c = x_c^B = 4.065 \times 10^6 \text{ MWh/year}$$

$$x_w = x_w^B = 3.326 \times 10^6 \text{ MWh/year}$$

Furthermore, we also obtain that  $\tilde{\delta}_w = 0.585$  and  $\tilde{\delta} = 0.466$ . As  $\delta$  increases from the threshold of  $\bar{\delta} = 0.442$ , the generation quantity from the wind farm will first increase then decrease. As  $\delta$  increases further from  $\tilde{\delta}_w = 0.585$ , the generation quantity from the wind energy will become even smaller than that of the non-binding case. A similar analysis can be conducted for the total generation quantity. In this case, the RPS policy will only be effective for a level from 0.442 to 0.466, in which the generation quantity from the wind farm and the total generation quantity will both increase relative to the case of no RPS. Figure 8 below summarizes the observation.

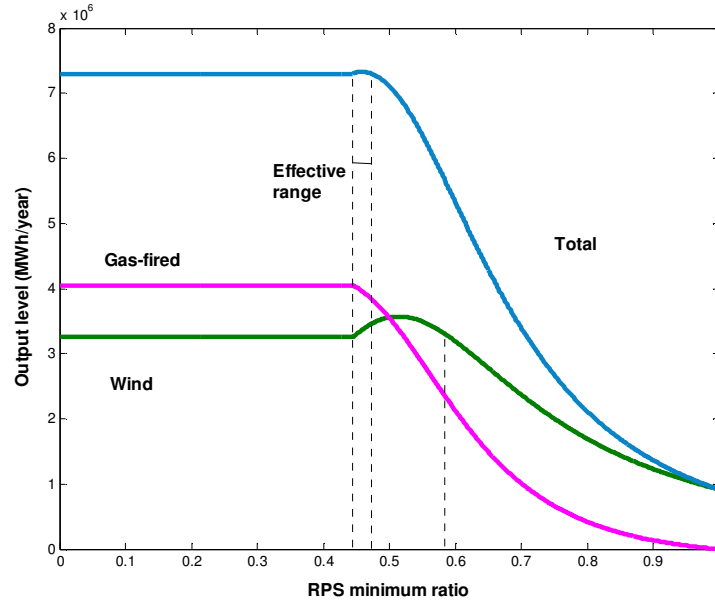


Figure 8. Impact of  $\delta$  in the numerical example

### 5.3 Impact of $\sigma_S$ , $\sigma_F$ and $\rho_{FS}$

First we investigate the impact of  $\sigma_S$  and  $\sigma_F$  assuming  $\delta = 0.2$ . From the previously given data, we have  $\beta_1 = -5.045$ ,  $\beta_2 = -25.685$  and  $\alpha_1 = 20.703$ . Hence we have the following results:

For  $\sigma_S$ , the binding solution is optimal when  $\sigma_S \geq 23.764$ , while the non-binding solution is optimal when  $0 < \sigma_S < 23.764$ .

For  $\sigma_F$ , the binding solution is optimal when  $0 < \sigma_F \leq 0.655$ , while the non-binding solution is optimal when  $\sigma_F > 0.655$ .

Figure 9 and Figure 10 illustrate the impact of  $\sigma_S$  and  $\sigma_F$  on the optimal generation portfolio. We note that, for the  $\sigma_F$  case, the threshold value of  $\sigma_F^* = 0.310$  leads to the maximum generation quantities from both sources.

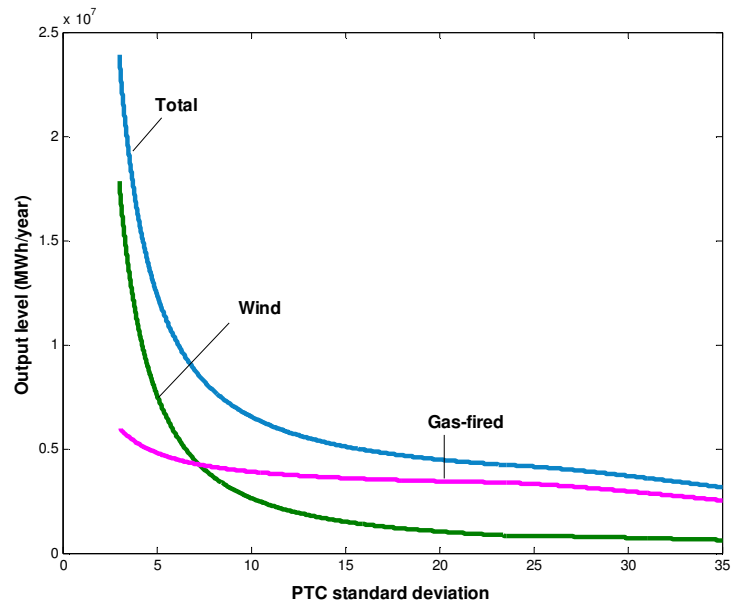


Figure 9. Impact of  $\sigma_S$  in the numerical example

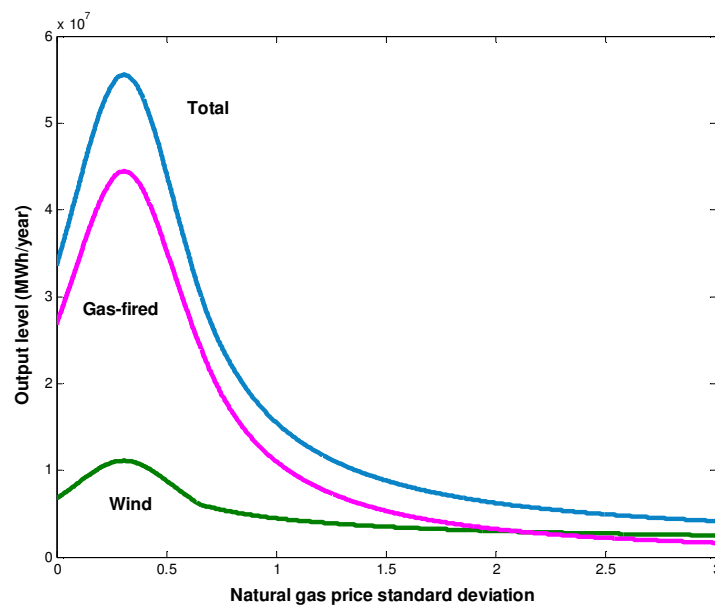


Figure 10. Impact of  $\sigma_F$  in the numerical example

Finally, we investigate the impact of  $\rho_{FS}$  while keeping other parameters the same.

From the previously given data, we have  $\beta_3 = -70.858$  and the binding solution is optimal

for any  $0 < \rho_{FS} < 1$ . Figure 11 illustrates the impact of  $\rho_{FS}$  on the optimal generation portfolio.

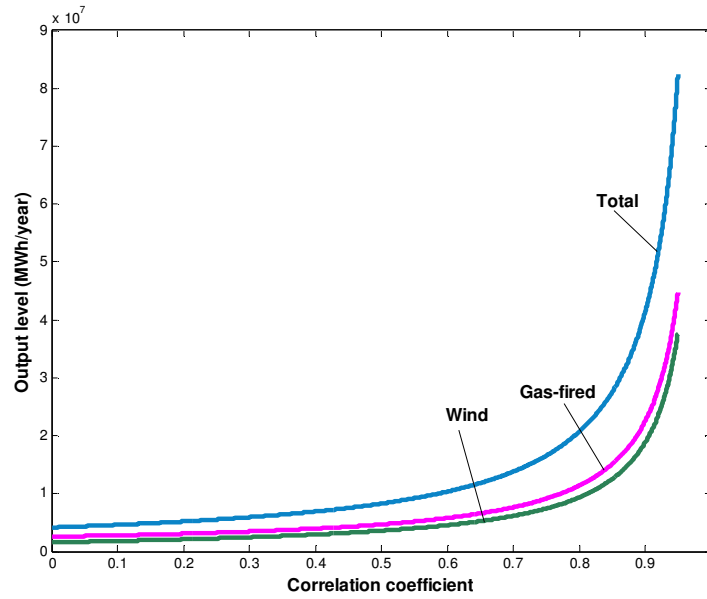


Figure 11. Impact of  $\rho_{FS}$  in the numerical example

## CHAPTER 6. POLICY IMPLICATIONS

Based on our investigation, there are a multiple number of critical policy implications. In this chapter, we first review the RPS background and goals in practice. Next, we briefly review the corresponding research findings, and provide relevant guidelines and recommendations on the RPS policy. We will next review the relevant research findings on the standard deviations and the correlation coefficient, and provide further policy implications on the RPS as well as the federal PTC mentioned in Introduction.

Currently, 29 states and DC have adopted the RPS with some variations on targets, eligible resources, and administrative responsibilities (DSIRE, 2010). The stated goals of the RPS also vary from state to state in directions, scopes, and numbers. Among these goals, some of critical goals that are most relevant to our investigation are listed as follows:

(1) Meet the increasing demands and ensure an adequate supply (see e.g., IL Public Act, ME Revised Statue, MI Enrolled Senator Bill and NJ Statue);

(2) Increase the reliance on eligible renewable energy resources (see e.g. CA Public Utilities Code);

(3) Reduce the reliance on fossil fuels and improve the energy security (see e.g. CT Docket);

(4) Reduce GHG emissions and improve the environment and public health (see e.g. CA Public Utilities Code).

As for the corresponding research findings, we recall from the parametric analysis on the RPS that, when  $G - (W + S) > 0$ , relative to the case of no RPS, if the goal of the RPS is to increase the generation quantity from the wind energy AND the total generation quantity,

the RPS level should be in the range of  $\bar{\delta} < \delta < \tilde{\delta}$ . Under this goal, if  $\delta \leq \bar{\delta}$ , the RPS is ineffective because it has no effect on the producer's generation decisions. If  $\delta \geq \tilde{\delta}$ , the RPS is ineffective because the total generation quantity does not increase. On the other hand, if a state's goal is only to increase the generation quantity from the wind power, then  $\delta$  should be in  $(\bar{\delta}, \tilde{\delta}_w)$ . Finally, if a state's goal is to decrease generation from the conventional source and reduce emissions, then  $\delta$  should be strictly greater than  $\bar{\delta}$ .

Meanwhile, when  $G - (W + S) \leq 0$ , we recall that the total generation quantity  $x^B$  decreases as  $\delta$  increases in  $(\bar{\delta}, 1)$ . This indicates that, relative to the case of no RPS, the goal of increasing the generation quantity from the wind energy AND the total generation quantity cannot be met with any  $\delta$  in  $(\bar{\delta}, 1)$ . On the other hand, if a state's goal is only to increase the generation quantity from the wind energy or to decrease generation from the conventional source and reduce emissions, then the ranges of  $\delta$  remain the same as in the case of  $G - (W + S) > 0$ .

By comparing the listed goals of the RPS and the corresponding research findings, we now present the following policy guidelines and recommendations.

(a) For states (e.g., Illinois, Maine, Michigan, and New Jersey) with a goal of ensuring an adequate supply of electric power and meeting the increasing demands relative to the case of no RPS,

a.1: if  $G - (W + S) > 0$ , an RPS level where  $\bar{\delta} < \delta < \tilde{\delta}$  is recommended.

a.2: if  $G - (W + S) \leq 0$ , no RPS level is effective.

(b) For states (e.g., California) with a goal of increasing the reliance on eligible renewable energy resources relative to the case of no RPS, an RPS level where  $\bar{\delta} < \delta < \tilde{\delta}_w$  is recommended.

(c) For states (e.g., Connecticut) with a goal of decreasing the reliance on fossil fuels as well as with a goal of reducing emissions relative to the case of no RPS, an RPS level where  $\delta > \bar{\delta}$  is recommended.

(d) From (a), (b), and (c), before setting an RPS level, it is recommended for the government to correctly estimate the current values of the critical economic parameters as the effectiveness of an RPS level is conditional upon a multiple number of critical parameter values.

(e) From (a), (b), and (c), we note that the current format of the implementation via a single RPS level is often inadequate. e.g., it may meet some goals, but not others; achieving a higher level of a goal may mean achieving a lower level of another goal concurrently. Hence, it is recommended that the government explore more sophisticated RPS mechanisms as a single PRS level may seem “primitive” and “blunt.”

Let us now proceed to review the relevant findings and recommendations with respect to the standard deviations and the correlation coefficient as follows. First, we recall from the parametric analysis on  $\sigma_s$  that, under the RPS, a lower degree of uncertainty in the wind PTC level will increase the generation quantities from both the conventional source and wind energy whether the solution is binding or not. This implies that, by reducing the uncertainty in the PTC level (e.g., make the PTC level more transparent and predictable according to the producer’s perception), the federal government could encourage the achievement of the goals



(1) and (2) of the RPS, but at the same time, the achievement of the goals (3) and (4) will become more difficult.

Next, we recall from the parametric analysis on  $\sigma_F$  that, under the RPS, when the solution is binding, a higher degree of uncertainty in the conventional fuel price does not necessarily discourage generation from any source (a higher degree of uncertainty within the range of  $(0, \sigma_F^*)$  will encourage generation quantities from both the conventional source and wind energy). When  $\sigma_F$  is sufficiently large ( $\sigma_F > \sigma_F^*$ ), the generation quantities from the conventional source and wind energy decrease whether the solution is binding or not. This will facilitate the achievement of the goals (3) and (4) of the RPS, but at the same time, this will discourage the achievement of the goal (1) and (2).

Finally, we recall from the parametric analysis on  $\rho_{FS}$  that, under the RPS, a higher level of correlation between the conventional fuel price and wind PTC level will encourage generation from both the conventional source and wind energy whether the solution is binding or not. Hence by strengthening the correlation between the conventional fuel price and wind PTC level (e.g., make this subsidy level even higher when the conventional fuel price increases), the federal government could encourage the achievement of the goal (1) and (2) of the RPS, but, at the same time, could discourage the achievement of the goals (3) and (4).

## CHAPTER 7. DISCUSSION OF NEGATIVE CORRELATION

In Chapter 3, it is presumed that the correlation coefficient between the conventional fuel price and the wind power PTC level is positive (see Fundamental Model Assumption 4). This presumption is justified based on our observation from both quantitative and qualitative aspects, as we elaborated before. On the other hand, however, we also recognize that, in the future, the uncertainty in the PTC level might also bring negative correlation between it and the fuel price. Therefore, it is also worthwhile for us to examine the model under the negative  $\rho_{FS}$ , which will also bring some interesting and relevant managerial insights.

We note that Technical Assumptions 1 through 3 still hold when  $\rho_{FS} < 0$ . Hence, it is easy to obtain that the optimal solutions and corresponding conditions on each parameter (except  $\rho_{FS}$  itself) are the same as those for the case of  $\rho_{FS} > 0$  (see Table 2 through Table 4).

Due to the mathematical structure of our model, to conducting comprehensive parametric analyses under negative  $\rho_{FS}$  are much more difficult (especially for  $\delta$  and  $\rho_{FS}$ ) and are still IN PROGRESS. In this chapter we aim to briefly discuss the results we obtained from the initial investigation and illustrate their relevant policy implications.

### 7.1 Results from the Preliminary Parametric Analyses

In this section we present the results we obtained from the preliminary parametric analyses on the RPS level  $\delta$ , standard deviations  $\sigma_S$  and  $\sigma_F$ , and correlation coefficient  $\rho_{FS}$ . We note that the comprehensive analyses are still in progress.

### 7.1.1 Result of the RPS Level $\delta$

Given  $\rho_{FS} < 0$ , we are able to obtain the following possible cases of the shapes of the optimal generation quantities from the conventional fuel source and the wind energy as well as the total generation quantity, i.e.,  $x_c^B$ ,  $x_w^B$ ,  $x^B$ ,  $x_c^{NB}$ ,  $x_w^{NB}$ , and  $x^{NB}$ . We note that when  $\delta = \bar{\delta}$ , there will be two distinctive slopes for each target at that point of  $\bar{\delta}$ . In the direction of a decrease, the results will be the same as  $0 < \delta < \bar{\delta}$ . On the other hand, in the direction of an increase, the results will be the same as when  $\bar{\delta} < \delta < 1$ .

(a) For the optimal generation quantity from the conventional fuel source, when  $\bar{\delta} < \delta < 1$ , there are two possible cases. Namely,

a.1:  $\frac{\partial x_c^B}{\partial \delta}$  is positive-negative in  $(\bar{\delta}, 1)$ .

a.2:  $\frac{\partial x_c^B}{\partial \delta}$  is negative in  $(\bar{\delta}, 1)$ .

When the P1 is non-binding optimal (i.e.,  $0 < \delta < \bar{\delta}$ ), it is easy to verify from (3.25) that  $\frac{\partial x_c^{NB}}{\partial \delta} = 0$ .

(b) For the optimal generation quantity from the wind energy, when  $\bar{\delta} < \delta < 1$ , there are also two possible cases. Namely,

b.1:  $\frac{\partial x_w^B}{\partial \delta}$  is positive-negative in  $(\bar{\delta}, 1)$ .

b.2:  $\frac{\partial x_w^B}{\partial \delta}$  is positive in  $(\bar{\delta}, 1)$ .

When the P1 is non-binding optimal (i.e.,  $0 < \delta < \bar{\delta}$ ), it is easy to verify from (3.26) that  $\frac{\partial x_w^{NB}}{\partial \delta} = 0$ .

(c) For the optimal total generation quantity, when  $\bar{\delta} < \delta < 1$ , there are three possible cases. Namely,

c.1:  $\frac{\partial x^B}{\partial \delta}$  is positive-negative.

c.2:  $\frac{\partial x^B}{\partial \delta}$  is negative.

c.3:  $\frac{\partial x^B}{\partial \delta}$  is positive.

When the P1 is non-binding optimal (i.e.,  $0 < \delta < \bar{\delta}$ ), it is easy to verify from (3.29) that  $\frac{\partial x_w^{NB}}{\partial \delta} = 0$ .

Due to the mathematical structure, to specify the corresponding conditions for each case is difficult. Here we use Figure 12 and Figure 13 from our numerical simulation to briefly illustrate two typical patterns which will provide us more managerial insights. Figure 12 shows under certain conditions, the generation quantity from the wind energy and the total generation quantity are always increasing, while that from the conventional source is decreasing when the RPS level increases from the threshold of  $\bar{\delta}$ .

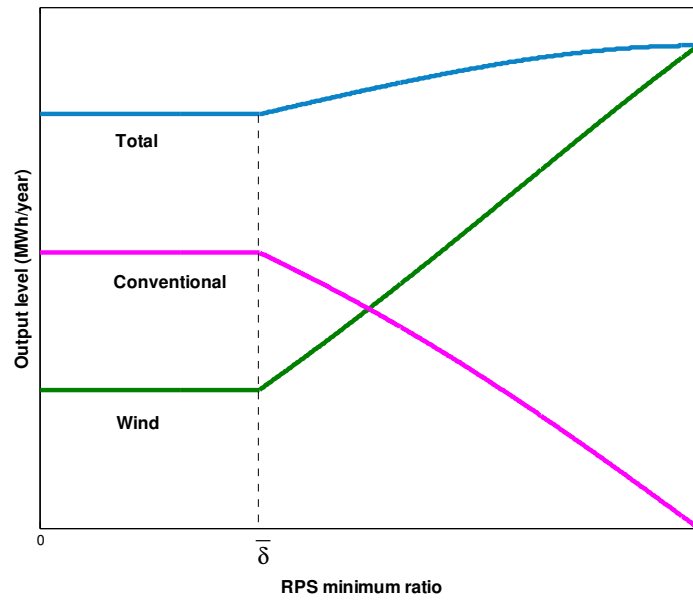


Figure 12. Possible case 1 of  $\delta$  on optimal generation portfolio given  $\rho_{FS} < 0$

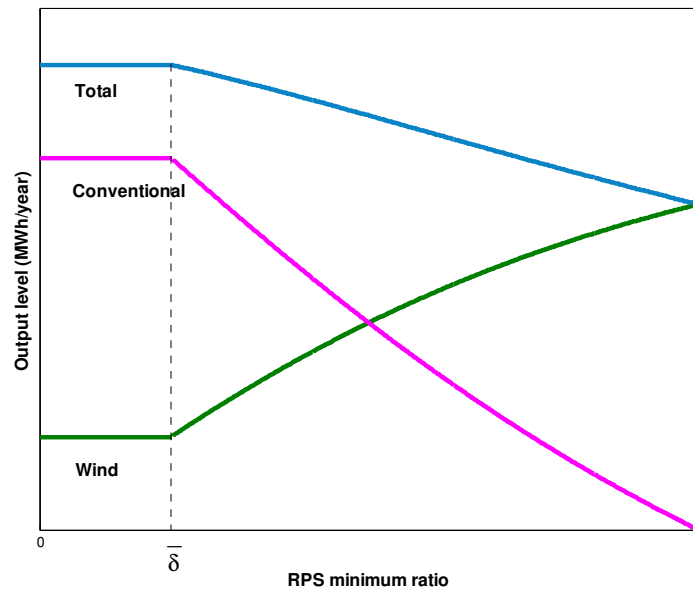


Figure 13. Possible case 2 of  $\delta$  on optimal generation portfolio given  $\rho_{FS} < 0$

Figure 13 provides another interesting case that the generation quantity from the wind energy is increasing, while that from the conventional source and the total generation quantity are both decreasing when the RPS level increases from the threshold of  $\bar{\delta}$ .

### 7.1.2 Result of the Standard Deviation $\sigma_s$

Given  $\rho_{FS} < 0$ , we are able to obtain the following possible cases of the shapes of  $x_c^B$ ,  $x_w^B$ ,  $x^B$ ,  $x_c^{NB}$ ,  $x_w^{NB}$ , and  $x^{NB}$ . We note that when  $\sigma_s = \bar{\sigma}_s$ , there will be two distinctive slopes at that point of  $\bar{\sigma}_s$ . In the direction of a decrease, the results will be the same as  $0 < \sigma_s < \bar{\sigma}_s$ . On the other hand, in the direction of an increase, the results will be the same as when  $\sigma_s > \bar{\sigma}_s$ .

(a) For the optimal generation quantity from the conventional fuel source, when  $\sigma_s > \bar{\sigma}_s$ ,  $\frac{\partial x_c^B}{\partial \sigma_s}$  is negative. When P1 is non-binding optimal (i.e.,  $0 < \sigma_s < \bar{\sigma}_s$ ),  $\frac{\partial x_c^{NB}}{\partial \sigma_s}$  is positive.

(b) For the optimal generation quantity from the wind energy, when  $\sigma_s > \bar{\sigma}_s$ ,  $\frac{\partial x_w^B}{\partial \sigma_s}$  is negative. When P1 is non-binding optimal (i.e.,  $0 < \sigma_s < \bar{\sigma}_s$ ),  $\frac{\partial x_c^{NB}}{\partial \sigma_s}$  is also negative.

(c) For the optimal total generation quantity, when  $\sigma_s > \bar{\sigma}_s$ ,  $\frac{\partial x^B}{\partial \sigma_s}$  is negative. When P1 is non-binding optimal (i.e.,  $0 < \sigma_s < \bar{\sigma}_s$ ), there are three possible cases. Namely,

$$c.1: \text{ If } 0 < \frac{-(W+S)\beta_1 + 2(W+S)\alpha}{G\beta_1} < \bar{\sigma}_s,$$

$$\frac{\partial x^{NB}}{\partial \sigma_s} \text{ is negative when } 0 < \sigma_s \leq \frac{-(W+S)\beta_1 + 2(W+S)\alpha}{G\beta_1};$$

$$\frac{\partial x^{NB}}{\partial \sigma_s} \text{ is positive when } \frac{-(W+S)\beta_1 + 2(W+S)\alpha}{G\beta_1} < \sigma_s < \bar{\sigma}_s.$$

$$\text{c.2: If } \frac{-(W+S)\beta_1 + 2(W+S)\alpha}{G\beta_1} \geq \bar{\sigma}_s, \frac{\partial x^{NB}}{\partial \sigma_s} \text{ is negative.}$$

$$\text{c.3: If } \frac{-(W+S)\beta_1 + 2(W+S)\alpha}{G\beta_1} \leq 0, \frac{\partial x^{NB}}{\partial \sigma_s} \text{ is positive.}$$

These three cases of  $\frac{\partial x^{NB}}{\partial \sigma_s}$  indicate that when the uncertainty of the PTC level is sufficiently small within the range of non-binding optimality, the effect of its increment on the total generation quantity may vary under a negative  $\rho_{FS}$ . Here we use Figure 14 from our numerical simulation to briefly illustrate the one of the typical pattern.

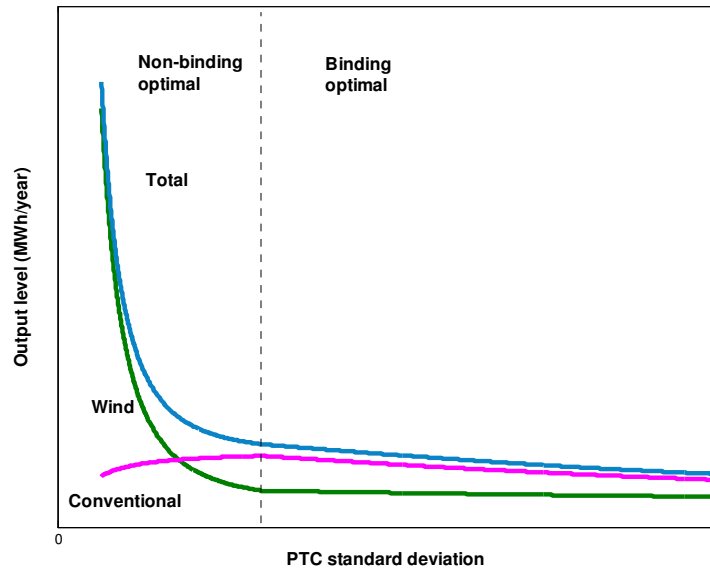


Figure 14. Possible case of  $\sigma_s$  on optimal generation portfolio given  $\rho_{FS} < 0$

From Figure 14, we note that given  $\rho_{FS} < 0$ , when  $\sigma_S$  is sufficiently small (i.e.,  $0 < \sigma_S < \bar{\sigma}_S$ ), an increment in  $\sigma_S$  will encourage the generation from the conventional source, which is different from the conclusion we obtained before for the case of positive  $\rho_{FS}$ .

### 7.1.3 Result of the Standard Deviation $\sigma_F$

Given  $\rho_{FS} < 0$ , we are able to obtain the following possible cases of the shapes of  $x_c^B$ ,  $x_w^B$ ,  $x^B$ ,  $x_c^{NB}$ ,  $x_w^{NB}$ , and  $x^{NB}$ . We note that when  $\sigma_F = \bar{\sigma}_F$ , there will be two distinctive slopes at that point of  $\bar{\sigma}_F$ . In the direction of an increase, the results will be the same as  $\sigma_F > \bar{\sigma}_F$ . On the other hand, in the direction of a decrease, the results will be the same as when  $0 < \sigma_F < \bar{\sigma}_F$ .

(a) For the optimal generation quantity from the conventional source, when  $0 < \sigma_F < \bar{\sigma}_F$ ,  $\frac{\partial x_c^B}{\partial \sigma_F}$  is negative. When P1 is non-binding optimal (i.e.,  $\sigma_F > \bar{\sigma}_F$ ),  $\frac{\partial x_c^{NB}}{\partial \sigma_F}$  is also negative.

(b) For the optimal generation quantity from the wind energy, when  $0 < \sigma_F < \bar{\sigma}_F$ ,  $\frac{\partial x_w^B}{\partial \sigma_F}$  is negative. When P1 is non-binding optimal (i.e.,  $\sigma_F > \bar{\sigma}_F$ ),  $\frac{\partial x_w^{NB}}{\partial \sigma_F}$  is positive.

(c) For the optimal total generation quantity, when  $0 < \sigma_F < \bar{\sigma}_F$ ,  $\frac{\partial x^B}{\partial \sigma_F}$  is negative.

When P1 is non-binding optimal (i.e.,  $\sigma_F > \bar{\sigma}_F$ ), there are two possible cases. Namely,

c.1: If  $\frac{2G\gamma - G\beta_2}{(W+S)\beta_2} \leq \bar{\sigma}_F$ ,  $\frac{\partial x^{NB}}{\partial \sigma_F}$  is positive.



$$c.2: \text{ If } \frac{2G\gamma - G\beta_2}{(W + S)\beta_2} > \bar{\sigma}_F,$$

$$\frac{\partial x^{NB}}{\partial \sigma_F} \text{ is negative when } \bar{\sigma}_F < \sigma_F \leq \frac{2G\gamma - G\beta_2}{(W + S)\beta_2};$$

$$\frac{\partial x^{NB}}{\partial \sigma_F} \text{ is positive when } \sigma_F > \frac{2G\gamma - G\beta_2}{(W + S)\beta_2}.$$

These two cases of  $\frac{\partial x^{NB}}{\partial \sigma_F}$  indicates that when the uncertainty of the fuel price is sufficiently large within the range of non-binding optimality, the effect of its increment on the total generation quantity may vary. Figure 15 below illustrates one of the typical patterns.

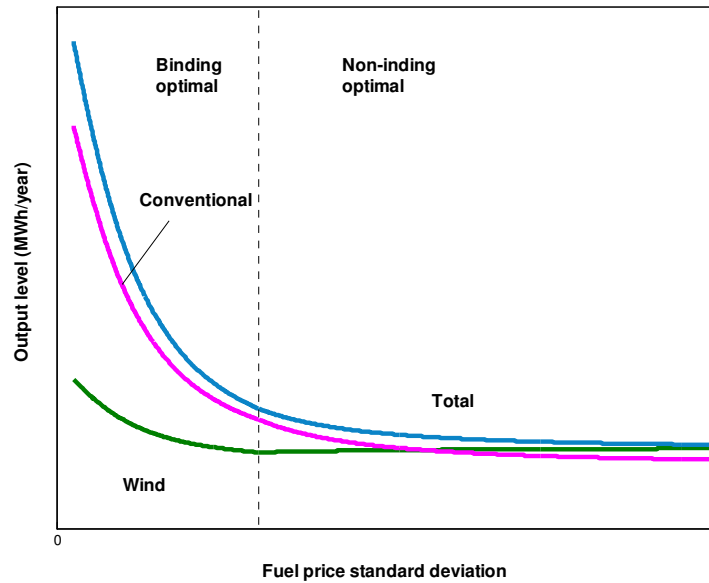


Figure 15. Possible case of  $\sigma_F$  on optimal generation portfolio given  $\rho_{FS} < 0$

From Figure 15, we note that given  $\rho_{FS} < 0$ , when  $\sigma_F$  is sufficiently large (i.e.,  $\sigma_F > \bar{\sigma}_F$ ), an increment in  $\sigma_F$  will encourage the generation from the wind energy, which is different from the conclusion we obtained before for the case of positive  $\rho_{FS}$ .

#### 7.1.4 Result of the Correlation Coefficient $\rho_{FS}$

Given  $\rho_{FS} < 0$ , first we have to re-derive the optimal solutions and corresponding conditions on  $\rho_{FS}$ . The derived approach is similar to what we did to for the case of positive  $\rho_{FS}$  in Section 4.5. The result is summarized in the following table.

**Table 10. Optimal conditions on  $\rho_{FS}$  given  $\rho_{FS} < 0$**

| $\rho_{FS}$   | Binding<br>optimality condition       | Non-binding<br>optimality condition |
|---|---------------------------------------|-------------------------------------|
| $(1-\delta)G - \delta(W+S) < 0$<br>$-1 < \bar{\rho}_{FS} < 0$   | $\bar{\rho}_{FS} \leq \rho_{FS} < 0$  | $-1 < \rho_{FS} < \bar{\rho}_{FS}$  |
| $(1-\delta)G - \delta(W+S) > 0$<br>$-1 < \bar{\rho}_{FS} < 0$   | $-1 < \rho_{FS} \leq \bar{\rho}_{FS}$ | $\bar{\rho}_{FS} < \rho_{FS} < 0$   |
| $(1-\delta)G - \delta(W+S) = 0$ ,<br>$(1-\delta)(W+S)\alpha - \delta G\gamma \leq 0$<br>or $(1-\delta)G - \delta(W+S) < 0$ ,<br>$\bar{\rho}_{FS} \leq -1$<br>or $(1-\delta)G - \delta(W+S) > 0$ ,<br>$\bar{\rho}_{FS} \geq 0$ | any $-1 < \rho_{FS} < 0$              | /                                   |
| $(1-\delta)G - \delta(W+S) = 0$ ,<br>$(1-\delta)(W+S)\alpha - \delta G\gamma > 0$   | /                                     | any $-1 < \rho_{FS} < 0$            |

|  |  |  |
|--|--|--|
| or $(1 - \delta)G - \delta(W + S) < 0$ ,<br>$\bar{\rho}_{FS} \geq 0$   |  |  |
| or $(1 - \delta)G - \delta(W + S) > 0$ ,<br>$\bar{\rho}_{FS} \leq -1$  |  |  |
| $\bar{\rho}_{FS} = \frac{(1 - \delta)(W + S)\alpha - \delta G \gamma}{[(1 - \delta)G - \delta(W + S)]\beta_3}$ |  |  |

Now we are able to obtain the following possible cases of the shapes of  $x_c^B$ ,  $x_w^B$ ,  $x^B$ ,  $x_c^{NB}$ ,  $x_w^{NB}$ , and  $x^{NB}$ . We note that for those conditions where  $\bar{\rho}_{FS}$  is applicable there will be two distinctive slopes at that point of  $\bar{\rho}_{FS}$ , which equals to the value when  $\bar{\rho}_{FS} \leq \rho_{FS} < 0$  and  $-1 < \rho_{FS} < \bar{\rho}_{FS}$ .

(a) For the optimal generation quantity from the conventional source, when P1 is binding optimal,  $\frac{\partial x_c^B}{\partial \rho_{FS}}$  is positive. When it is non-binding optimal, there are two possible cases. Namely,

a.1:  $\frac{\partial x_c^{NB}}{\partial \rho_{FS}}$  is positive.

a.2:  $\frac{\partial x_c^{NB}}{\partial \rho_{FS}}$  is negative-positive.

(b) For the optimal generation quantity from the wind energy, when P1 is binding optimal,  $\frac{\partial x_w^B}{\partial \rho_{FS}}$  is positive. When it is non-binding optimal, there are also two possible cases.

Namely,

b.1:  $\frac{\partial x_w^{NB}}{\partial \rho_{FS}}$  is positive.

b.2:  $\frac{\partial x_w^{NB}}{\partial \rho_{FS}}$  is negative-positive.

(c) For the optimal total generation quantity, when P1 is binding optimal,  $\frac{\partial x^B}{\partial \rho_{FS}}$  is positive. When it is non-binding optimal, there are also two possible cases. Namely,

c.1:  $\frac{\partial x^{NB}}{\partial \rho_{FS}}$  is positive.

c.2:  $\frac{\partial x^{NB}}{\partial \rho_{FS}}$  is negative-positive.

Due to the mathematical structure, to specify the corresponding conditions for each case is difficult. Here we use Figure 16 below from our numerical simulation to briefly illustrate one of the typical patterns which will provide us more managerial insights.

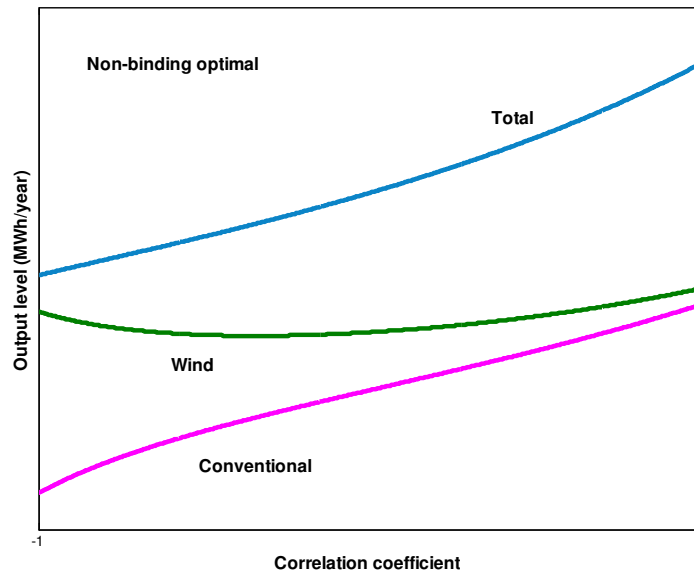


Figure 16. Possible case of  $\rho_{FS}$  on optimal generation portfolio given  $\rho_{FS} < 0$

From Figure 16, we note that, given  $\rho_{FS} < 0$ , a higher level of  $\rho_{FS}$  does not necessarily increase the generation quantity from the wind energy, which is different from the conclusion we obtained for the case of positive  $\rho_{FS}$ .

## 7.2 Preliminary Policy Implications

Based on the results of preliminary analyses, we briefly summarize the policy implications when  $\rho_{FS} < 0$ .

First of all, recall Figure 12, a higher level of the RPS can always encourage the generation from the wind energy and the total generation, while discourage the generation from the conventional source. This implies that in this particular case, a single scalar of the RPS in  $(\bar{\delta}, 1)$  is sufficient to meet all the goals listed in Chapter 6; the higher  $\delta$  is, the better that the goals could be met. However, this is not always true. For example, Figure 13 shows that under some circumstance, current format of the implementation via a single RPS level is inadequate: it may meet some goals, but not others; achieving a higher level of a goal may mean achieving a lower level of another goal concurrently.

Second, we recall from the analysis of  $\sigma_s$  in 7.1.2 that, under the RPS, when the optimal solution is non-binding, a lower degree of uncertainty in the PTC level always increases the generation quantity from the wind energy, but does not necessarily increase that from the conventional source and the total generation quantity. This implies that, when  $\rho_{FS} < 0$ , the federal government might be able to encourage the achievement of all the goals of the RPS at the same time by reducing the uncertainty in the PTC level.

Third, we recall from the analysis on  $\sigma_F$  in 7.1.3 that, under the RPS, when the optimal solution is non-binding, a higher degree of uncertainty in fuel price always discourages the generation from the conventional source, but does not necessarily discourage that from the wind energy and the total generation. This implies that, when  $\rho_{FS} < 0$ , increasing the uncertainty level of the fuel price might facilitate the achievement of all the goals of the RPS.

Finally, we recall from the analysis on  $\rho_{FS}$  in 7.1.4 that, under the RPS, when the optimal solution is non-binding, a higher level of correlation between the conventional fuel price and wind PTC level does not necessarily encourage the generation from both source. Instead, a higher level of  $\rho_{FS}$  might decrease the generation quantity of some source or the total generation quantity. Hence, strengthening the correlation between the conventional fuel price and wind PTC level is not always a good way to encourage the wind energy development.

## CHAPTER 8. CONCLUSION

In this thesis we formulated and analyzed a mean-variance utility maximization model for a risk-averse electric power generation company who wishes to determine the optimal levels of capacity and generation for a single conventional source and the wind energy under RPS regulation where the conventional fuel price and wind power PTC level are random variables. This study is motivated by a strong desire to reconcile competing claims for the roles of the RPS on generation expansion planning and the stochastic nature of the PTC level.

Throughout our intensive parametric analyses, we were able to obtain a multiple number of relevant and interesting managerial insights and economic implications when the conventional fuel price and wind power PTC are positively correlated (which is justified and consistent with our observation). For example, we showed that a higher level of the RPS does not necessarily encourage generation (or capacity expansion) from the wind energy relative to the case of no RPS. Under certain circumstances, the need to introduce the RPS is reduced. This also indicates that given a single level of RPS, it is possible to help achieve some goals and go against some other goals. Hence, some refinement in RPS may be necessary. We also find an “effective range” of the RPS level that will increase the generation quantity from the wind energy and the total generation quantity while decrease the generation quantity from the conventional source at the same time.

Second, we showed that a lower degree of uncertainty in the wind power PTC level will increase the generation quantities (or capacity levels) from both the conventional source and the wind energy. This is intuitively consistent with the producer’s risk-averse nature.

Third, we revealed an interesting fact that a higher degree of uncertainty in the conventional fuel price does not necessarily discourage generation (or capacity expansion) from any source. Instead, within a certain range, a bigger  $\sigma_f$  leads to a higher level of generation (or capacity expansion) from both sources. This conclusion opposes the common statement that increasing uncertainty in the conventional fuel price discourages electricity generation.

Forth, we showed a higher level of correlation between the conventional fuel price and wind PTC level will encourage generation quantities from both the conventional source and wind energy.

Finally, based on all the findings, a list of policy guidelines and recommendations was presented which could be used to improve the effectiveness of the RPS and wind power PTC corresponding to their stated goals.

In addition, we also realized the worth of conducting similar analyses when the fuel price and wind power PTC are negatively correlated. Through our preliminary analyses we were also able to provide some valuable results as well as policy implications.

While it is still too early to fully assess the effectiveness of RPS adopted by states, the formulation and analyses in this paper can serve as a basis for numerous future studies. Based on the model, the effect of the PTC could be understood better if we investigate the case when the uncertainty in the PTC is approaching zero. Moreover, as specifically mentioned in Chapter 3, some simplifying assumptions can be relaxed. For example, what if the electricity selling price is also a random variable? More random variable will bring in a more complicated variance structure, with more correlation terms. Such a relaxation effort will widen the applicability of our model. Another example would be to introduce more



generating candidates and physical constraints, which develops our model into a large-scale GEP problem. Advanced algorithms might be needed to solve this kind of problem efficiently from a numerical perspective. Dynamic modeling methods such as Binomial Lattice, Geometric Brownian Motion also need to be explored. In addition, as more and better sets of data become available on the PTC and RPS, a fully empirical study (cf. an analytical study with a numerical example) on generation expansion planning seems to be worthwhile.

## APPENDIX

Here we briefly give the outlines of verification of Proposition 1, 2 and 3 in Chapter 4.

### A.1 Verification of Proposition 1

By rewriting the binding optimality condition (3.24) it is easy to get the first statement proved, i.e.,  $\frac{\partial U^B}{\partial \delta} \leq 0$ .

For  $\frac{\partial x_c^B}{\partial \delta}$ , by following the investigation process we present in Chapter 4 we have the two roots when  $-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma \neq 0$  as

$$\frac{-2G\beta + G\gamma + (W + S)\alpha \pm \sqrt{G\gamma[G\gamma - (W + S)\beta] + (W + S)\gamma[(W + S)\alpha - G\beta]}}{-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma}$$

For simplification we denote the two roots as  $r_1$  (with smaller numerator) and  $r_2$ .

In order to grasp the derivative's property, we follow the steps summarized in the beginning of this section and compare both  $r_1$  and  $r_2$  with the boundary of binding optimal, i.e.,  $\bar{\delta}$  and 1.

$$r_1 - 1 = \frac{-[G - (W + S)]\gamma - \sqrt{\gamma[G^2\gamma - 2G(W + S)\beta + (W + S)^2\alpha]}}{-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma}$$

$$r_2 - 1 = \frac{-[G - (W + S)]\gamma + \sqrt{\gamma[G^2\gamma - 2G(W + S)\beta + (W + S)^2\alpha]}}{-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma}$$

$$r_1 - \bar{\delta} = \frac{\left( \begin{array}{l} (\gamma - \beta) [G^2\gamma - 2G(W + S)\beta + (W + S)^2\alpha] \\ - [(W + S)\alpha - G\beta + G\gamma - (W + S)\beta] \times \sqrt{\gamma [G^2\gamma - 2G(W + S)\beta + (W + S)^2\alpha]} \end{array} \right)}{(-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma) [(W + S)\alpha - G\beta + G\gamma - (W + S)\beta]}$$

$$r_2 - \bar{\delta} = \frac{\left( \begin{array}{l} (\gamma - \beta) [G^2\gamma - 2G(W + S)\beta + (W + S)^2\alpha] \\ + [(W + S)\alpha - G\beta + G\gamma - (W + S)\beta] \times \sqrt{\gamma [G^2\gamma - 2G(W + S)\beta + (W + S)^2\alpha]} \end{array} \right)}{(-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma) [(W + S)\alpha - G\beta + G\gamma - (W + S)\beta]}$$

For  $r_1$  there are five cases:  $r_1 > 1$ ,  $r_1 = 1$ ,  $\bar{\delta} < r_1 < 1$ ,  $r_1 = \bar{\delta}$ ,  $r_1 < \bar{\delta}$ ;

For  $r_2$  there are five cases:  $r_2 > 1$ ,  $r_2 = 1$ ,  $\bar{\delta} < r_2 < 1$ ,  $r_2 = \bar{\delta}$ ,  $r_2 < \bar{\delta}$ .

Thus we have  $5 \times 5 = 25$  cases of  $r_1$  and  $r_2$ . The proof of the statement ii in Proposition 1 can be obtained through proving the following three lemmas.

**Lemma 1** If  $\rho_{FS} > 0$ ,  $G > 0$ , and  $W + S > 0$  as implied by Fundamental Model

Assumption 4, Technical Assumptions 2 and 3, then  $\frac{\partial x_c^B}{\partial \delta}$  is negative in  $(\bar{\delta}, 1)$  when

$$-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma > 0.$$

We show the proof outlines as follows:

When  $-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma > 0$ ,  $r_1 < r_2$ .

The condition for  $r_2 - 1 > 0$  to hold is

$$\sqrt{\gamma [G^2\gamma - 2G(W + S)\beta + (W + S)^2\alpha]} > [G - (W + S)]\gamma$$

a. when  $G - (W + S) \leq 0$ , it always holds;

b. when  $G - (W + S) > 0$ , squaring both sides we have

$$G^2\gamma^2 - 2G(W + S)\beta\gamma + (W + S)^2\alpha\gamma > G^2\gamma^2 + (W + S)^2\gamma^2 - 2G(W + S)\gamma^2$$

After simplifying we have

$$(W + S)\gamma[-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma] > 0$$

Since  $W + S > 0$ ,  $\gamma > 0$ , we have  $-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma > 0$ .

Therefore when  $-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma > 0$ , the inequality will always hold, i.e.,  $r_2 - 1 > 0$ .

Similarly, we consider the condition for  $r_1 - \bar{\delta} > 0$  to hold is

$$\begin{aligned} & (\gamma - \beta)[G^2\gamma - 2G(W + S)\beta + (W + S)^2\alpha] \\ & > [(W + S)\alpha - G\beta + G\gamma - (W + S)\beta] \times \sqrt{\gamma[G^2\gamma - 2G(W + S)\beta + (W + S)^2\alpha]} \end{aligned}$$

Since  $\gamma - \beta > 0$ , dividing both sides by  $\sqrt{G^2\gamma - 2G(W + S)\beta + (W + S)^2\alpha}$  we have

$$(\gamma - \beta)\sqrt{G^2\gamma - 2G(W + S)\beta + (W + S)^2\alpha} > [(W + S)\alpha - G\beta + G\gamma - (W + S)\beta]\sqrt{\gamma}$$

Squaring both sides we have

$$(\gamma - \beta)^2 [G^2\gamma - 2G(W + S)\beta + (W + S)^2\alpha] > [(W + S)\alpha - G\beta + G\gamma - (W + S)\beta]^2 \gamma$$

After simplifying we have

$$(\alpha\gamma - \beta^2)(W + S)[(W + S)\gamma - 2G\gamma + 2G\beta - (W + S)\alpha] > 0$$

Since  $\alpha\gamma - \beta^2$ ,  $W + S > 0$ , we have  $(W + S)\gamma - 2G\gamma + 2G\beta - (W + S)\alpha > 0$  which contradicts to  $-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma > 0$ .

Therefore when  $-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma > 0$ , the inequality will never hold, i.e.,  $r_1 - \bar{\delta} < 0$ .

Summarizing what we obtained we have the conclusion that given  $\rho_{FS} > 0$ , when  $-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma > 0$ , we have the bigger root  $r_2 > 1$  while the smaller root  $r_1 < \bar{\delta}$  which indicates  $\frac{\partial x_c^B}{\partial \delta}$  is negative in  $(\bar{\delta}, 1)$ .

**Lemma 2** If  $\rho_{FS} > 0$ ,  $G > 0$ , and  $W + S > 0$  as implied by Fundamental Model Assumption 4, Technical Assumptions 2 and 3, then  $\frac{\partial x_c^B}{\partial \delta}$  is negative in  $(\bar{\delta}, 1)$  when  $-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma < 0$ .

Similar to the proof of Lemma 1, we are able to obtain that when  $-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma < 0$ , the bigger root  $r_1 < \bar{\delta}$  which indicates  $\frac{\partial x_c^B}{\partial \delta}$  is negative in  $(\bar{\delta}, 1)$ .

**Lemma 3** If  $\rho_{FS} > 0$ ,  $G > 0$ , and  $W + S > 0$  as implied by Fundamental Model Assumption 4, Technical Assumptions 2 and 3, then  $\frac{\partial x_c^B}{\partial \delta}$  is negative in  $(\bar{\delta}, 1)$  when  $-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma = 0$ .

The proof outlines are as follows:

When  $-2G\beta + 2G\gamma + (W + S)\alpha - (W + S)\gamma = 0$ , we have

$$\frac{\partial x_c^B}{\partial \delta} = \frac{[4G\beta - 2G\gamma - 2(W + S)\alpha]\delta - 2G\beta + (W + S)\alpha}{R[(1 - \delta)^2\alpha + 2\delta(1 - \delta)\beta + \delta^2\gamma]^2}$$

Since the numerator is a linear function, we can obtain the trend of it by examining the boundary of  $\delta = \bar{\delta}$  and 1.

a. at  $\delta = \bar{\delta}$

$$\begin{aligned}\frac{\partial x_c^B}{\partial \delta} \Big|_{\delta=\bar{\delta}} &= \frac{[4G\beta - 2G\gamma - 2(W+S)\alpha]\bar{\delta} - 2G\beta + (W+S)\alpha}{R[(1-\delta)^2\alpha + 2\delta(1-\delta)\beta + \delta^2\gamma]^2} \\ &= \frac{[(W+S)\alpha - G\beta][-(W+S)(\alpha + \beta) + 2G\beta] - G(W+S)(\alpha\gamma - \beta^2)}{R[(1-\delta)^2\alpha + 2\delta(1-\delta)\beta + \delta^2\gamma]^2 [(W+S)\alpha - G\beta + G\gamma - (W+S)\beta]}\end{aligned}$$

From the equality we have  $W+S = \frac{2G(\gamma - \beta)}{\gamma - \alpha}$ . Substituting  $W+S$  with  $\frac{2G(\gamma - \beta)}{\gamma - \alpha}$

we have the numerator as  $\frac{G^2[-2\alpha(\gamma - \beta) + \beta(\gamma - \alpha) - (\gamma - \beta)](\alpha\gamma - \beta^2)}{(\gamma - \alpha)}$ .

Since  $\gamma - \alpha > 0$  and  $-2\alpha(\gamma - \beta) + \beta(\gamma - \alpha) - (\gamma - \beta) < 0$ , we have  $\frac{\partial x_c^B}{\partial \delta} \Big|_{\delta=\bar{\delta}} < 0$ .

b. at  $\delta = 1$

$$\begin{aligned}\frac{\partial x_c^B}{\partial \delta} \Big|_{\delta=1} &= \frac{[4G\beta - 2G\gamma - 2(W+S)\alpha] - 2G\beta + (W+S)\alpha}{R[(1-\delta)^2\alpha + 2\delta(1-\delta)\beta + \delta^2\gamma]^2} \\ &= \frac{2G\beta - 2G\gamma - (W+S)\alpha}{R[(1-\delta)^2\alpha + 2\delta(1-\delta)\beta + \delta^2\gamma]^2} < 0\end{aligned}$$

Therefore, when  $-2G\beta + 2G\gamma + (W+S)\alpha - (W+S)\gamma = 0$ ,  $\frac{\partial x_c^B}{\partial \delta} < 0$  for  $(\bar{\delta}, 1)$ .

Hereby we proved the statement ii of Proposition 1. Statement iii and iv can also be proved in similar approaches.

## A.2 Verification of Proposition 2

The proof of Proposition 2 is briefly outlined as follows:

By observation we know the necessary and sufficient condition for the four inequalities to hold is  $(1-\delta)\beta_1 + \delta\sigma_s > 0$ .

We note that we already have the condition under which is problem is binding optimal that  $\sigma_s \geq \bar{\sigma}_s$ , thus when  $\sigma_s \geq \bar{\sigma}_s$ ,

$$\begin{aligned}
& (1-\delta)\beta_1 + \delta\sigma_s \\
& \geq (1-\delta)\beta_1 + \delta\bar{\sigma}_s \\
& = \frac{[(1-\delta)G + \delta(W+S)]\beta_1 + \sqrt{[(1-\delta)G - \delta(W+S)]^2 \beta_1^2 + 4\delta(1-\delta)G(W+S)\alpha}}{2G} \\
& = \frac{[(1-\delta)G + \delta(W+S)]\beta_1 + \sqrt{[(1-\delta)G + \delta(W+S)]^2 \beta_1^2 + 4\delta(1-\delta)G(W+S)(\alpha - \beta_1^2)}}{2G}
\end{aligned}$$

Recall that  $(\alpha - \beta_1^2) > 0$ , we have the formula above greater than zero, i.e.,  $(1-\delta)\beta_1 + \delta\bar{\sigma}_s > 0$ . Therefore,  $(1-\delta)\beta_1 + \delta\sigma_s > 0$  when  $\sigma_s > \bar{\sigma}_s$ .

### A.3 Verification of Proposition 3

The proof of Proposition 3 is briefly outlined as follows:

By observation we know we need to examine the property of  $(1-\delta)\alpha_1\sigma_F + \delta\beta_2$ , which is the determinant of the derivatives' properties. The sign of the derivation will be the opposite as that of  $(1-\delta)\alpha_1\sigma_F + \delta\beta_2$ .

Since the determinant expression is a linear function, we can obtain the sign of the determinant by checking the zero point and  $\bar{\sigma}_F$ .

$$(1-\delta)\alpha_1\sigma_F + \delta\beta_2 \Big|_{\sigma_F=0} = \delta\beta_2 < 0$$

$$\begin{aligned}
& (1-\delta)\alpha_1\sigma_F + \delta\beta_2 \Big|_{\sigma_F=\bar{\sigma}_F} \\
& = \frac{[(1-\delta)G + \delta(W+S)]\beta_2 + \sqrt{[(1-\delta)G - \delta(W+S)]^2 \beta_2^2 + 4\delta(1-\delta)G(W+S)\alpha_1\gamma}}{2(W+S)} \\
& = \frac{[(1-\delta)G + \delta(W+S)]\beta_2 + \sqrt{[(1-\delta)G + \delta(W+S)]^2 \beta_2^2 + 4\delta(1-\delta)G(W+S)(\alpha_1\gamma - \beta_2^2)}}{2(W+S)}
\end{aligned}$$

Recall that  $(\alpha_1\gamma - \beta_2^2) > 0$ , we have  $(1-\delta)\alpha_1\sigma_F + \delta\beta_2|_{\sigma_F=\bar{\sigma}_F} > 0$ . Thus we know  $(1-\delta)\alpha_1\sigma_F + \delta\beta_2$  will be first negative and then positive in  $(0, \bar{\sigma}_F)$ , which indicates that

$\frac{\partial x_c^B}{\partial \sigma_F}$ ,  $\frac{\partial x_w^B}{\partial \sigma_F}$ ,  $\frac{\partial x^B}{\partial \sigma_F}$ , and  $\frac{\partial U^B}{\partial \sigma_F}$  will first be positive and then negative when  $0 < \sigma_F < \bar{\sigma}_F$ .



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